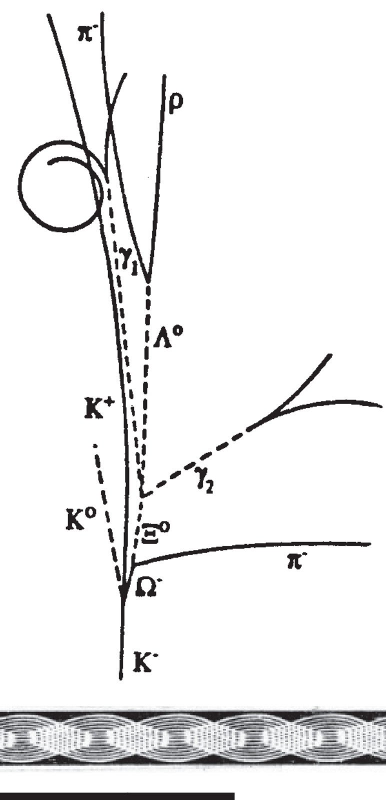
VOLUME 48, NUMBER 1, MARCH 2025

HADRONIC JOURNAL Volume 48, Number 1, March 2025

HADRONIC JOURNAL

Founded in 1978 by Prof. R. M. Santilli at Harvard University. Some of the past Editors include Professors S. L. Adler, A. 0. Barut, L. C. Biedenharn, N. N. Bogoliubov, M. Froissart, J.Lohmus, S. Okubo, Nobel Laureate Ilya Prigogine, M. L. Tomber, J. P.Vigier, J. Wess, Nobel Laureate Chen Ning Yang.



HADRONIC PRESS, INC.

HADRONIC JOURNAL

Established in 1978 by Prof. R. M. Santilli at Harvard University Hadronic Journal and Algebras, Groups and Geometries have been regularly published since 1978 without publication charges and are among the few remaining independent refereed journals.

This Journal publishes advances research papers and Ph. D. theses in any field of mathematics without publication charges.

For subscription, format and any other information please visit the website http://www.hadronicpress.com

HADRONIC PRESS INC.
35246 U. S. 19 North Suite 215
Palm Harbor, FL 34684, U.S.A.
http://www.hadronicpress.com
Email: info@hadronicpress.com
Phone: +1-727-946-0427

HADRONIC JOURNAL

VOLUME 48, NUMBER 1, MARCH 2025

HADRONIC JOURNAL 48, 1 - 61 (2025)
DOI 10.29083/HJ.48.01.2025/SC1

UNDERSTANDING SUPER HEAVY STABLE MASS NUMBERS AND MAXIMUM BINDING ENERGY OF ANY MASS NUMBER WITH REVISED STRONG AND ELECTROWEAK MASS FORMULA

U. V. S. Seshavatharam
Honorary faculty, I-SERVE, Survey no-42, Hitech City
Hyderabad-84, Telangana, India
seshavatharam.uvs@gmail.com
S. Lakshminarayana
Department of Nuclear Physics, Andhra University
Visakhapatnam-03, AP, India
lnsrirama@gmail.com

Received December 3, 2024

Abstract

With reference to currently believed Semi Empirical Mass Formula (SEMF), our "Strong and Electroweak Mass Formula' (SEWMF) constitutes 4 simple terms and only one energy coefficient of magnitude 10.1 MeV. First term is a volume term, second term seems to be a representation of free nucleons associated with electroweak interaction, third term is a radial term and fourth one is an asymmetry term about the mean stable mass number. In this paper, we make an attempt to understand and estimate the maximum binding energy associated with any mass number. For A > 4, $(BE)_A \cong \{A - 0.000935A^2 - A^{1/3} - A^{-1/2}\}10.1$ MeV. We are working on refining the 4th term with even-odd corrections, shell corrections and other microscopic corrections. Stable mass numbers and super heavy mass numbers can be understood with a relation of the form, $A_s \cong \left[\text{RoundOff}\left\{(Z+2.9464)^{1.2} - 1.7165\right\} + [0,1]\right] \pm 2n$ where $n \cong 0,1,2.$ It needs a review with respect to even-odd proton numbers and other microscopic corrections. Based on the concept of "binding energy per nucleon", the most complicated Avogadro number and Unified atomic mass unit can be estimated in a unified approach.

Keywords: Semi Empirical Mass Formula (SEMF); Strong and Electroweak Mass Formula (SEWMF); free nucleons; light house like stable mass number; super heavy mass numbers; revised electroweak term; maximum binding energy of any mass number; Avogadro number; Unified atomic mass unit;

1. Introduction

Based on 4G model of final unification, in our recent publications [1-10], we have clearly shown that, strong and weak interactions, play a vital role in basic nuclear structure. With our strong and electroweak mass formula, nuclear binding energy can be estimated with one unified energy coefficient having 4 simple terms. In this paper, considering our contribution pertaining to DAE-BRNS 2023 symposium proceedings [1] and recent journal publication [2], we make a minor change for understanding the maximum binding energy associated with any mass number. It can be refined with further study. Proceeding further, we discuss on estimating stable mass numbers having relatively long living time. In this context, considering proton number as an input, we have developed a simple power law applicable for the whole range of periodic table. Based on the data fit for lower, medium and heavy stable nuclides, we are sure to say that, it will certainly help in designing and developing new experimental set ups for producing stable and unstable super heavy atomic nuclides. With further research, atomic nuclides suitable for safe nuclear energy production and suitable for medical applications can be produced with ease and confidence. Proceeding further, Unified atomic mass unit and Avogadro number can be estimated in a unified approach.

2. Three assumptions of 4G model of final Unification

Following our 4G model of final unification [1-10]

1) There exists a characteristic electroweak fermion of rest energy, $M_{wf}c^2 \cong 584.725$ GeV. It can be considered as the zygote of all elementary particles.
2) There exists a nuclear elementary charge in such a way that, $\left(\frac{e}{e_n}\right)^2 \cong \alpha_s \cong 0.1152 =$ Strong coupling constant [11] and $e_n \cong 2.9464e$.
3) Each atomic interaction is associated with a characteristic large gravitational coupling constant. Their fitted magnitudes are,

$G_e \cong$ Electromgnetic gravitational constant $\cong 2.374335 \times 10^{37}$ m^{3}kg^{-1}sec^{-2}

$G_n \cong$ Nuclear gravitational constant $\cong 3.329561 \times 10^{28}$m^{3}kg^{-1}sec^{-2}

$G_w \cong$ Electroweak gravitational constant $\cong 2.909745 \times 10^{22}$ m^{3}kg^{-1}sec^{-2}

It may be noted that,

1) Weak interaction point of view, following our assumptions, Fermi's weak coupling constant [11] can be fitted with the following relations.

$$\left.\begin{array}{l} G_F \cong \left(\dfrac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong G_w M_{wf}^2 R_w^2 \cong 1.44021\times10^{-62}\ \text{J.m}^3 \\ \text{where,} \begin{cases} R_0 \cong \dfrac{2G_n m_p}{c^2} \cong 1.24\ \times10^{-15}\ \text{m} \\ R_w \cong \dfrac{2G_w M_{wf}}{c^2} \cong 6.75\times10^{-19}\ \text{m} \end{cases} \end{array}\right\} \tag{1}$$

2) In a unified approach, most important point to be noted is that, $\hbar c \cong G_w M_{wf}^2$. Clearly speaking, based on the electroweak interaction, the well believed quantum constant $\hbar c$ seems to have a deep inner meaning [10]. It needs further study with respect to condensed matter physics.

3. Free nucleons and the Electroweak term

With reference to our strong and electroweak mass formula [1-10],

3) Nuclear volume can be split into 'core inner' and 'core outer'.
4) Nucleons residing in nuclear inner core help in increasing nuclear binding energy.
5) Nucleons residing in outer core will not involve in nuclear binding.
6) Outer core nucleons can be called as free or electroweak nucleons.
7) Proportionality coefficient being $\dfrac{m_p}{M_{wf}} \cong \dfrac{938.272\ \text{MeV}}{584725\ \text{MeV}} \cong 0.001605,$ free nucleon number is proportional to half of the sum of squared proton number and squared mass number.

8) Considering light and heavy atomic nuclides, by considering a correction factor $\left[2-\left(\frac{N}{Z}\right)\right]$, in our recent publications, we expressed our first approximate relation for free nucleon number as, $A_{free} \cong \left[2-\left(\frac{N}{Z}\right)\right]+\left[0.0016\left(\frac{Z^2+A^2}{2}\right)\right] \cong \left[2-\left(\frac{N}{Z}\right)\right]+\left[0.0008\left(Z^2+A^2\right)\right]$.
9) For medium and heavy proton numbers and their isotopes, equality of excess neutron number and free nucleon number can be considered as an index of possible stability. It needs a review at fundamental level.

4. Nuclear radius and Radial term

1) Interesting observation is that, nuclear binding energy seems to decrease with increasing radius.
2) As nuclear volume is proportional the mass number, it is possible to understand the decreasing nuclear binding energy with cube root of the mass number $A_{rad} \cong A^{1/3}$.

5. Stable mass number and Asymmetry term

1) Even though it is not exact stable mass number, we understood that, the ratio of nuclear charge and elementary charge and electroweak interaction seem to play a crucial role in understanding and estimating the approximate stable mass number of any atomic nuclide having a proton number Z. This is one best practical application of our proposed nuclear charge and electroweak fermion.
2) Stable mass number seems to play a crucial role in estimating the binding energy of other isotopes of Z.
3) Our estimated mass number close to stability can be called as 'light house (like) mass number' where one can find the beginning of relatively long living isotopes of Z.
4) Keeping light and heavy atomic nuclides in view, we suggest a common and simple relation of the form [2,6],

$$\left.\begin{aligned} A_s &\cong \text{RoundOff}\left\{\left(Z+\left(\frac{e_n}{e}\right)\right)^{1.2}-\sqrt{\frac{e_n}{e}}\right\} \\ &\cong \text{RoundOff}\left\{(Z+2.9464)^{1.2}-1.7165\right\} \\ \text{where } &\left(\frac{e_n}{e}\right)^{\frac{1}{6}} \cong \left(\frac{1}{\alpha_s}\right)^{\frac{1}{12}} \cong 1.19733 \cong 1.2 \end{aligned}\right] \tag{2A}$$

It may be noted that, right selection of stable mass number greatly helps in minimizing the error in estimating nuclear binding energy. Especially, for light atomic nuclides, whose stable mass number is very close to 2Z, estimated binding energy seems to be on lower side compared to actual binding energy. Hence, it seems better to select stable mass number of Z based on their relative time of living. Considering even-odd corrections, above relation can be refined for a better understanding in the following way. It can be reviewed in a better way with further study.

1) If Z is even and obtained A_s is odd, then, $A_s \cong A_s + 1$.
2) If Z is even and obtained A_s is even, then, $A_s \cong A_s$.
3) If Z is odd and obtained A_s is odd, then, $A_s \cong A_s$.
4) If Z is odd and obtained A_s is even, then, $A_s \cong A_s + 1$.

See Table 1 for a better understanding.

$$A_s \cong \text{RoundOff}\left\{(Z+2.9464)^{1.2}-1.7165\right\} + \text{EO correction} \cong [0,1] \tag{2B}$$

Following this relation, for odd elements, their best possible three mass numbers can be expressed as,

$$\begin{aligned} A_s &\cong \left[\text{RoundOff}\left\{(Z+2.9464)^{1.2}-1.7165\right\}+[0,1]\right]+2n \\ &\text{where } n = 0,1,2 \end{aligned} \tag{2C}$$

We are working on the possibility of considering $(-2n)$ for increasing the estimated range of heavy mass numbers on lower side [12-16]. Thus, relation (2C) can be expressed as,

$$A_s \cong \left[\text{RoundOff}\left\{(Z+2.9464)^{1.2} - 1.7165\right\} + [0,1]\right] \pm 2n \qquad \text{(2D)}$$
$$\text{where } n = 0,1,2$$

In Table 1, we have presented the estimated light house like mass numbers of Z = 6 to 118.

Table 1: Estimated light house like stable mass numbers of Z=5 to118

Proton number	Estimated stable mass number	Estimated mass number with EO corrections		Proton number	Estimated stable mass number	Estimated mass number with EO corrections		Proton number	Estimated stable mass number	Estimated mass number with EO corrections
5	10	11		43	97	97		81	202	203
6	12	12		44	100	100		82	205	206
7	14	15		45	102	103		83	208	209
8	16	16		46	105	106		84	211	212
9	18	19		47	107	107		85	214	215
10	20	20		48	110	110		86	217	218
11	22	23		49	113	113		87	219	219
12	24	24		50	115	116		88	222	222
13	26	27		51	118	119		89	225	225
14	28	28		52	121	122		90	228	228
15	30	31		53	123	123		91	231	231
16	32	32		54	126	126		92	234	234
17	35	35		55	129	129		93	237	237
18	37	38		56	131	132		94	240	240
19	39	39		57	134	135		95	243	243
20	41	42		58	137	138		96	246	246
21	43	43		59	140	141		97	249	249
22	46	46		60	142	142		98	252	252
23	48	49		61	145	145		99	255	255
24	50	50		62	148	148		100	258	258
25	53	53		63	151	151		101	261	261
26	55	56		64	153	154		102	264	264

27	57	57		65	156	157		103	268	269
28	60	60		66	159	160		104	271	272
29	62	63		67	162	163		105	274	275
30	65	66		68	165	166		106	277	278
31	67	67		69	167	167		107	280	281
32	69	70		70	170	170		108	283	284
33	72	73		71	173	173		109	286	287
34	74	74		72	176	176		110	289	290
35	77	77		73	179	179		111	292	293
36	79	80		74	182	182		112	295	296
37	82	83		75	185	185		113	298	299
38	84	84		76	187	188		114	301	302
39	87	87		77	190	191		115	304	305
40	89	90		78	193	194		116	308	308
41	92	93		79	196	197		117	311	311
42	94	94		80	199	200		118	314	314

Following the above points, super heavy atomic nuclides' possible mass number range can be understood. Here we would like to highlight the point that compared to other mass numbers, these mass numbers can have relatively long living time. It needs further study, experimental design, set up and observations. For example,

1) Mass numbers of Z=100: 254 to 262
2) Mass numbers of Z=101: 257 to 265
3) Mass numbers of Z=102: 260 to 268
4) Mass numbers of Z=103: 265 to 273
5) Mass numbers of Z=104: 268 to 276
6) Mass numbers of Z=105: 271 to 279
7) Mass numbers of Z=106: 274 to 282
8) Mass numbers of Z=107: 277 to 285
9) Mass numbers of Z=108: 280 to 288
10) Mass numbers of Z=109: 283 to 291
11) Mass numbers of Z=110: 286 to 294
12) Mass numbers of Z=111: 289 to 297

13) Mass numbers of Z=112: 292 to 300
14) Mass numbers of Z=113: 295 to 303
15) Mass numbers of Z=114: 298 to 306
16) Mass numbers of Z=115: 302 to 309
17) Mass numbers of Z=116: 304 to 312
18) Mass numbers of Z=117: 307 to 315
19) Mass numbers of Z=118: 310 to 318

For the case of currently believed heavy magic proton number, Z=114, its estimated mass number range is 298 to 306. Here it is quite interesting to note that, lower mass number is 298 and its corresponding neutron number is 298-114=184 and is nicely matching with the currently believed heavy magic neutron number at 184. Thus, Z=114 and A=298 can be given a strong priority for its experimental identification as a doubly magic super heavy atomic nuclide [16].

5) Number 0.0016 plays a very interesting role in estimating the free nucleon number as,

$$A_{free} \cong \left[2-\left(\frac{N}{Z}\right)\right]+0.0016\left[\left(Z^2+N^2+\left(\frac{Z^2}{N}\right)^2\right)-ZN\left(\frac{N-Z}{N+Z}\right)^2\right]$$
$$\cong \left[2-\left(\frac{N}{Z}\right)\right]+0.0016\left[\left(Z^2+N^2+\left(\frac{Z^2}{N}\right)^2\right)-ZN\left(\frac{A-2Z}{A}\right)^2\right] \quad (3)$$

where $\left[2-\left(\frac{N}{Z}\right)\right]$ is a correction factor that needs a review.

6) Here, very interesting point to be noted is that, the number 0.0016 can also be understood as a ratio of the mean mass of pions to the mean mass of electroweak bosons. It can be expreessed as,

$$\frac{m_p}{M_{wf}} \cong \left(\frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_z c^2)^0 (m_w c^2)^\pm}}\right) \cong \left(\frac{\sqrt{134.98\times 139.57}\text{ MeV}}{\sqrt{80379.0\times 91187.6}\text{ MeV}}\right) \cong 0.0016032 \quad (4)$$

Based on this unique observation, we are very confident to say that, strong and weak interactions play a vital role exploring the secrets of nuclear structure.

7) Independent of proton number, approximate asymmetry term can be expressed as,

$$A_{asym} \cong \frac{(A_s - A)^2}{A_s} \tag{5}$$

It may be noted that, even though it is an approximate relation, it greatly helps in estimating the binding energy of isotopes for the entire range of atomic nuclides. It seems essential to work on this kind of relations.

6. Unique binding energy coefficient

We would like to emphasize the point that, nuclear binding energy can be understood with only one fixed energy coefficient. It can be understood in two different ways as expressed in following way.

Considering Up and Down quark masses [11],

$$\begin{aligned} B_0 &\cong \frac{1}{2}\left[\left(2m_uc^2 + m_dc^2\right) + \left(m_uc^2 + 2m_dc^2\right)\right] \\ &\cong \frac{3}{2}\left(m_uc^2 + m_dc^2\right) \cong 10.1 \text{ MeV (Our fit)} \\ &\text{where } \begin{cases} m_u \cong 2.16^{+0.49}_{-0.26} \text{ MeV}/c^2 \\ m_d \cong 4.67^{+0.48}_{-0.17} \text{ MeV}/c^2 \end{cases} \end{aligned} \tag{6}$$

Considering strong coupling constant and reduced Compton wavelength of proton,

$$\begin{aligned} &B_0 \cong -\left(\frac{1}{\sqrt{\alpha_s}}\right)\frac{e^2}{8\pi\varepsilon_0\left(\hbar/m_pc\right)} \cong -\frac{e_n^2}{8\pi\varepsilon_0\left(G_nm_p/c^2\right)} \cong -10.1 \text{ MeV} \\ &\text{where} \begin{cases} \alpha_s = \text{Strong coupling constant} \cong 0.115 \text{ to } 0.12 \\ \hbar/m_pc = \text{Reduced Comption wavelength of proton} \\ G_nm_p/c^2 \cong 0.62\times10^{-15}\,\text{m} \end{cases} \end{aligned} \tag{7A}$$

Considering B_0 as a form of total energy, it is possible to define its corresponding potential energy as,

$$E_{pot} \cong -\frac{e_n^2}{4\pi\varepsilon_0\left(G_n m_p/c^2\right)} \cong -20.2 \text{ MeV} \tag{7B}$$

Using this energy unit, various energy coefficients of the currently beloved semi empirical mass can be fitted.

7. Revised and reference formulae for nuclear binding energy.

The most famous and most advanced SEMF that follows isospin concept can be expressed as [17-21],

$$BE \cong \left[\begin{array}{l} \left\{\left[1+\left(\frac{4k_v}{A^2}\right)|T_z|\left(|T_z|+1\right)\right]a_v * A\right\} + \left\{\left[1+\left(\frac{4k_s}{A^2}\right)|T_z|\left(|T_z|+1\right)\right]a_s * A^{\frac{2}{3}}\right\} \\ +\left\{a_c * \left(\frac{Z^2}{A^{1/3}}\right)\right\} + \left\{f_p * \frac{Z^2}{A}\right\} + E_p \end{array}\right] \tag{8}$$

where, $T_z \cong \text{3rd component of isospin} = \frac{1}{2}(Z-N)$

$$\left[\begin{array}{l} a_v = -15.4963 \text{ MeV} \cong -20.2(1-2\alpha_s) \cong -15.546 \text{ MeV} \\ a_s = 17.7937 \cong -20.2(1-\alpha_s) \cong -17.873 \text{ MeV} \quad \text{MeV} \\ k_v = -1.8232 \cong -\left[2-\left[\frac{(1+\alpha_s)}{(1-\alpha_s)}\right]^2 \alpha_s\right] \cong -(2-0.183) \cong -1.817 \\ k_s = -2.2593 \cong -\left[2+\left[\frac{(1+\alpha_s)}{(1-\alpha_s)}\right]^2 \alpha_s\right] \cong -(2+0.183) \cong -2.183 \\ a_c = 0.7093 \cong 0.71 \text{ MeV} \\ f_p = -1.2739 \text{ MeV} \cong -\frac{20.2\alpha_s}{2} \cong -1.1635 \text{ MeV} \\ \left.\begin{array}{l} d_n = 4.6919 \text{ MeV}, d_p = 4.7230 \text{ MeV} \\ d_n \cong d_p \cong 2 * 20.2\alpha_s \cong 4.6541 \text{ MeV} \end{array}\right\} \\ d_{np} = -6.4920 \text{ MeV} \cong -3 * 20.2\alpha_s \cong -6.981 \text{ MeV} \end{array}\right]$$

$$\text{and}\ \begin{cases} \text{for } (Z, N) \text{ Odd, } E_p \cong \dfrac{d_n}{N^{1/3}} + \dfrac{d_p}{Z^{1/3}} + \dfrac{d_{np}}{A^{2/3}} \\ \text{for (Odd } Z\text{, Even } N\text{), } E_p \cong \dfrac{d_p}{Z^{1/3}} \\ \text{for (Even } Z\text{, Odd } N\text{), } E_p \cong \dfrac{d_n}{N^{1/3}} \\ \text{for (Even } Z\text{, Even } N\text{), } E_p \cong 0 \end{cases}.$$

For Z=6 to 118, replacing the factor $\left[2-\left(\frac{N}{Z}\right)\right]$, by $\left(\frac{1}{2}\right)$, we express our revised binding energy relation as,

$$BE \cong \begin{Bmatrix} A - \left\{\left(\frac{1}{2}\right) + 0.0016\left[\left(Z^2 + N^2 + \left(\frac{Z^2}{N}\right)^2\right) - N^2\left(\frac{N-Z}{N+Z}\right)^2\right]\right\} \\ -A^{1/3} - \frac{(A_s - A)^2}{A_s} \end{Bmatrix} \cdot 10.1 \text{ MeV} \quad (9)$$

$$\text{where, } \sqrt{\left(\frac{N-Z}{N+Z}\right)^2} \cong \beta \ldots (say)$$

See Table 2. For the estimated binding energy of isotopes of Z=6 to 121 in steps of Z=6,11,16.... One can understand its best possible scope by considering Z=121 and A=399. Its estimated binding energy is 2432.9 MeV and reference binding energy is 2427.8 MeV.

Table 2. Estimated binding energy of isotopes of Z=6,11,16,121

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
6	8	2	-4	12	0.5000	1.1	36.2	37.6	1.4
6	9	3	-3	12	0.3333	0.8	54.2	49.1	-5.2
6	10	4	-2	12	0.2000	0.7	68.7	65.1	-3.6
6	11	5	-1	12	0.0909	0.7	80.9	73.3	-7.6
6	12	6	0	12	0.0000	0.7	91.3	85.4	-5.9
6	13	7	1	12	0.0769	0.7	99.9	90.7	-9.2
6	14	8	2	12	0.1429	0.7	106.7	99.7	-7.1
6	15	9	3	12	0.2000	0.7	111.9	102.7	-9.2
6	16	10	4	12	0.2500	0.7	115.3	109.2	-6.1
6	17	11	5	12	0.2941	0.8	117.1	110.5	-6.6
6	18	12	6	12	0.3333	0.8	117.2	115.1	-2.1

6	19	13	7	12	0.3684	0.8	115.6	114.9	-0.7
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
11	18	7	-4	23	0.2222	1.3	131.8	123.2	-8.6
11	19	8	-3	23	0.1579	1.2	146.2	139.5	-6.7
11	20	9	-2	23	0.1000	1.1	159.4	151.4	-8.0
11	21	10	-1	23	0.0476	1.1	171.5	164.7	-6.8
11	22	11	0	23	0.0000	1.1	182.5	174.2	-8.4
11	23	12	1	23	0.0435	1.1	192.6	185.1	-7.5
11	24	13	2	23	0.0833	1.1	201.7	192.4	-9.3
11	25	14	3	23	0.1200	1.1	209.9	201.3	-8.6
11	26	15	4	23	0.1539	1.2	217.1	206.9	-10.3
11	27	16	5	23	0.1852	1.2	223.5	214.0	-9.5
11	28	17	6	23	0.2143	1.2	228.9	218.1	-10.8
11	29	18	7	23	0.2414	1.3	233.4	223.7	-9.7
11	30	19	8	23	0.2667	1.3	237.0	226.6	-10.5
11	31	20	9	23	0.2903	1.3	239.8	231.0	-8.8
11	32	21	10	23	0.3125	1.4	241.6	232.7	-8.9
11	33	22	11	23	0.3333	1.4	242.5	236.0	-6.6
11	34	23	12	23	0.3529	1.5	242.6	236.7	-5.9
11	35	24	13	23	0.3714	1.5	241.8	239.1	-2.7
11	36	25	14	23	0.3889	1.6	240.1	239.0	-1.1
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
16	28	12	-4	32	0.1429	1.9	228.3	217.4	-10.9
16	29	13	-3	32	0.1035	1.8	240.9	230.0	-10.9
16	30	14	-2	32	0.0667	1.8	252.6	245.5	-7.1
16	31	15	-1	32	0.0323	1.7	263.5	256.1	-7.4
16	32	16	0	32	0.0000	1.7	273.7	269.5	-4.2
16	33	17	1	32	0.0303	1.7	283.1	278.3	-4.7
16	34	18	2	32	0.0588	1.8	291.7	289.9	-1.8
16	35	19	3	32	0.0857	1.8	299.7	297.2	-2.5
16	36	20	4	32	0.1111	1.8	307.0	307.2	0.2
16	37	21	5	32	0.1351	1.8	313.6	313.1	-0.5
16	38	22	6	32	0.1579	1.9	319.5	321.7	2.3
16	39	23	7	32	0.1795	1.9	324.7	326.4	1.7
16	40	24	8	32	0.2000	2.0	329.3	333.8	4.5
16	41	25	9	32	0.2195	2.0	333.2	337.4	4.2
16	42	26	10	32	0.2381	2.1	336.5	343.7	7.2
16	43	27	11	32	0.2558	2.1	339.1	346.3	7.3
16	44	28	12	32	0.2727	2.2	341.0	351.7	10.6
16	45	29	13	32	0.2889	2.3	342.3	353.4	11.1
16	46	30	14	32	0.3044	2.3	343.0	357.9	14.9
16	47	31	15	32	0.3192	2.4	343.0	358.9	15.9
16	48	32	16	32	0.3333	2.5	342.4	362.5	20.1
16	49	33	17	32	0.3469	2.5	341.1	362.8	21.7
16	50	34	18	32	0.3600	2.6	339.2	365.7	26.5
16	51	35	19	32	0.3726	2.7	336.6	365.4	28.8
16	52	36	20	32	0.3846	2.8	333.4	367.6	34.2
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
21	38	17	-4	43	0.1053	2.7	316.3	303.1	-13.2
21	39	18	-3	43	0.0769	2.7	328.8	318.9	-9.9
21	40	19	-2	43	0.0500	2.6	340.6	331.4	-9.2

21	41	20	-1	43	0.0244	2.6	351.8	345.5	-6.4
21	42	21	0	43	0.0000	2.6	362.4	356.4	-6.1
21	43	22	1	43	0.0233	2.6	372.4	368.8	-3.6
21	44	23	2	43	0.0455	2.6	381.9	378.3	-3.5
21	45	24	3	43	0.0667	2.7	390.7	389.4	-1.3
21	46	25	4	43	0.0870	2.7	399.1	397.6	-1.5
21	47	26	5	43	0.1064	2.7	406.9	407.4	0.5
21	48	27	6	43	0.1250	2.8	414.1	414.5	0.3
21	49	28	7	43	0.1429	2.8	420.9	423.1	2.2
21	50	29	8	43	0.1600	2.9	427.1	429.1	2.0
21	51	30	9	43	0.1765	3.0	432.9	436.8	3.9
21	52	31	10	43	0.1923	3.0	438.1	441.8	3.8
21	53	32	11	43	0.2076	3.1	442.8	448.5	5.7
21	54	33	12	43	0.2222	3.2	447.0	452.7	5.7
21	55	34	13	43	0.2364	3.2	450.7	458.5	7.8
21	56	35	14	43	0.2500	3.3	454.0	461.9	8.0
21	57	36	15	43	0.2632	3.4	456.7	467.0	10.3
21	58	37	16	43	0.2759	3.5	458.9	469.6	10.7
21	59	38	17	43	0.2881	3.5	460.7	473.9	13.2
21	60	39	18	43	0.3000	3.6	462.0	476.0	14.0
21	61	40	19	43	0.3115	3.7	462.8	479.6	16.8
21	62	41	20	43	0.3226	3.8	463.1	481.0	17.9
21	63	42	21	43	0.3333	3.9	462.9	484.0	21.1
21	64	43	22	43	0.3438	4.0	462.2	484.8	22.6
21	65	44	23	43	0.3539	4.1	461.0	487.3	26.2
21	66	45	24	43	0.3636	4.2	459.4	487.6	28.2
21	67	46	25	43	0.3731	4.3	457.3	489.5	32.2
21	68	47	26	43	0.3824	4.4	454.7	489.3	34.6
21	69	48	27	43	0.3913	4.5	451.6	490.7	39.1

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
26	48	22	-4	56	0.0833	3.9	397.6	392.0	-5.6
26	49	23	-3	56	0.0612	3.8	410.7	404.9	-5.8
26	50	24	-2	56	0.0400	3.8	423.2	420.4	-2.8
26	51	25	-1	56	0.0196	3.8	435.3	431.9	-3.3
26	52	26	0	56	0.0000	3.7	446.8	446.0	-0.8
26	53	27	1	56	0.0189	3.8	457.9	456.3	-1.6
26	54	28	2	56	0.0370	3.8	468.5	469.1	0.6
26	55	29	3	56	0.0546	3.8	478.6	478.2	-0.4
26	56	30	4	56	0.0714	3.8	488.3	489.8	1.5
26	57	31	5	56	0.0877	3.9	497.6	497.8	0.3
26	58	32	6	56	0.1035	3.9	506.4	508.4	1.9
26	59	33	7	56	0.1186	4.0	514.9	515.4	0.6
26	60	34	8	56	0.1333	4.0	522.9	525.0	2.1
26	61	35	9	56	0.1475	4.1	530.5	531.1	0.7
26	62	36	10	56	0.1613	4.2	537.7	539.8	2.1
26	63	37	11	56	0.1746	4.2	544.5	545.1	0.6
26	64	38	12	56	0.1875	4.3	550.9	552.9	2.0
26	65	39	13	56	0.2000	4.4	556.9	557.5	0.6
26	66	40	14	56	0.2121	4.5	562.5	564.4	2.0
26	67	41	15	56	0.2239	4.6	567.7	568.3	0.6
26	68	42	16	56	0.2353	4.7	572.5	574.6	2.0
26	69	43	17	56	0.2464	4.8	577.0	577.8	0.8
26	70	44	18	56	0.2571	4.9	581.0	583.3	2.3
26	71	45	19	56	0.2676	5.0	584.7	585.9	1.2
26	72	46	20	56	0.2778	5.1	588.0	590.9	2.9
26	73	47	21	56	0.2877	5.2	590.9	592.9	2.0

26	74	48	22	56	0.2973	5.3	593.4	597.3	3.8
26	75	49	23	56	0.3067	5.4	595.6	598.8	3.2
26	76	50	24	56	0.3158	5.5	597.4	602.5	5.2
26	77	51	25	56	0.3247	5.6	598.8	603.5	4.8
26	78	52	26	56	0.3333	5.7	599.8	606.8	7.0
26	79	53	27	56	0.3418	5.8	600.5	607.3	6.9
26	80	54	28	56	0.3500	5.9	600.7	610.1	9.4
26	81	55	29	56	0.3580	6.0	600.6	610.2	9.6
26	82	56	30	56	0.3659	6.2	600.2	612.6	12.4
26	83	57	31	56	0.3735	6.3	599.3	612.2	12.9
26	84	58	32	56	0.3810	6.4	598.1	614.1	16.0
26	85	59	33	56	0.3882	6.5	596.5	613.4	16.9
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
31	58	27	-4	67	0.0690	5.2	481.7	472.1	-9.6
31	59	28	-3	67	0.0509	5.2	494.7	487.9	-6.8
31	60	29	-2	67	0.0333	5.1	507.2	500.8	-6.4
31	61	30	-1	67	0.0164	5.1	519.2	515.2	-4.0
31	62	31	0	67	0.0000	5.1	530.8	527.0	-3.9
31	63	32	1	67	0.0159	5.1	542.0	540.3	-1.7
31	64	33	2	67	0.0313	5.1	552.8	550.9	-1.8
31	65	34	3	67	0.0462	5.2	563.2	563.2	0.0
31	66	35	4	67	0.0606	5.2	573.2	572.8	-0.3
31	67	36	5	67	0.0746	5.2	582.8	584.1	1.3
31	68	37	6	67	0.0882	5.3	592.0	592.8	0.8
31	69	38	7	67	0.1015	5.4	600.9	603.1	2.2
31	70	39	8	67	0.1143	5.4	609.4	611.0	1.6
31	71	40	9	67	0.1268	5.5	617.5	620.4	2.9
31	72	41	10	67	0.1389	5.6	625.3	627.5	2.1
31	73	42	11	67	0.1507	5.6	632.8	636.1	3.3
31	74	43	12	67	0.1622	5.7	639.9	642.4	2.5
31	75	44	13	67	0.1733	5.8	646.6	650.3	3.7
31	76	45	14	67	0.1842	5.9	653.1	655.9	2.8
31	77	46	15	67	0.1948	6.0	659.1	663.1	3.9
31	78	47	16	67	0.2051	6.1	664.9	668.0	3.1
31	79	48	17	67	0.2152	6.2	670.3	674.5	4.2
31	80	49	18	67	0.2250	6.3	675.4	678.9	3.5
31	81	50	19	67	0.2346	6.4	680.1	684.8	4.6
31	82	51	20	67	0.2439	6.5	684.6	688.5	4.0
31	83	52	21	67	0.2530	6.6	688.7	693.8	5.2
31	84	53	22	67	0.2619	6.8	692.4	697.0	4.6
31	85	54	23	67	0.2706	6.9	695.9	701.8	5.9
31	86	55	24	67	0.2791	7.0	699.0	704.5	5.5
31	87	56	25	67	0.2874	7.1	701.8	708.7	6.9
31	88	57	26	67	0.2955	7.2	704.3	711.0	6.6
31	89	58	27	67	0.3034	7.4	706.5	714.7	8.2
31	90	59	28	67	0.3111	7.5	708.3	716.5	8.1
31	91	60	29	67	0.3187	7.6	709.9	719.8	9.9
31	92	61	30	67	0.3261	7.8	711.1	721.1	10.0
31	93	62	31	67	0.3333	7.9	712.0	723.9	12.0
31	94	63	32	67	0.3404	8.0	712.5	724.9	12.3
31	95	64	33	67	0.3474	8.2	712.8	727.3	14.5
31	96	65	34	67	0.3542	8.3	712.8	727.8	15.1
31	97	66	35	67	0.3608	8.4	712.4	729.9	17.5
31	98	67	36	67	0.3674	8.6	711.7	730.0	18.3
31	99	68	37	67	0.3737	8.7	710.7	731.7	21.0
31	100	69	38	67	0.3800	8.9	709.4	731.5	22.1

31	101	70	39	67	0.3861	9.0	707.8	732.8	25.0
31	102	71	40	67	0.3922	9.2	705.9	732.3	26.5
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
36	68	32	-4	80	0.0588	6.8	558.4	554.2	-4.2
36	69	33	-3	80	0.0435	6.8	571.7	567.4	-4.3
36	70	34	-2	80	0.0286	6.8	584.6	583.1	-1.6
36	71	35	-1	80	0.0141	6.7	597.1	595.2	-1.9
36	72	36	0	80	0.0000	6.7	609.2	609.7	0.5
36	73	37	1	80	0.0137	6.7	621.0	620.9	-0.1
36	74	38	2	80	0.0270	6.7	632.3	634.4	2.1
36	75	39	3	80	0.0400	6.8	643.4	644.7	1.3
36	76	40	4	80	0.0526	6.8	654.1	657.2	3.2
36	77	41	5	80	0.0649	6.9	664.4	666.6	2.2
36	78	42	6	80	0.0769	6.9	674.4	678.3	3.9
36	79	43	7	80	0.0886	7.0	684.1	686.9	2.7
36	80	44	8	80	0.1000	7.0	693.5	697.7	4.2
36	81	45	9	80	0.1111	7.1	702.6	705.5	3.0
36	82	46	10	80	0.1220	7.2	711.3	715.6	4.3
36	83	47	11	80	0.1325	7.3	719.8	722.7	2.9
36	84	48	12	80	0.1429	7.4	727.9	732.0	4.1
36	85	49	13	80	0.1529	7.4	735.8	738.4	2.7
36	86	50	14	80	0.1628	7.5	743.3	747.1	3.8
36	87	51	15	80	0.1724	7.6	750.6	752.9	2.3
36	88	52	16	80	0.1818	7.8	757.5	760.8	3.3
36	89	53	17	80	0.1910	7.9	764.2	766.0	1.8
36	90	54	18	80	0.2000	8.0	770.6	773.4	2.8
36	91	55	19	80	0.2088	8.1	776.7	778.0	1.3
36	92	56	20	80	0.2174	8.2	782.5	784.8	2.3
36	93	57	21	80	0.2258	8.3	788.0	788.9	0.9
36	94	58	22	80	0.2340	8.5	793.3	795.1	1.9
36	95	59	23	80	0.2421	8.6	798.3	798.7	0.5
36	96	60	24	80	0.2500	8.7	803.0	804.4	1.5
36	97	61	25	80	0.2577	8.9	807.4	807.5	0.2
36	98	62	26	80	0.2653	9.0	811.5	812.8	1.2
36	99	63	27	80	0.2727	9.1	815.4	815.4	0.0
36	100	64	28	80	0.2800	9.3	819.0	820.2	1.2
36	101	65	29	80	0.2871	9.4	822.3	822.4	0.0
36	102	66	30	80	0.2941	9.6	825.4	826.7	1.3
36	103	67	31	80	0.3010	9.7	828.2	828.5	0.3
36	104	68	32	80	0.3077	9.9	830.7	832.4	1.7
36	105	69	33	80	0.3143	10.0	832.9	833.8	0.9
36	106	70	34	80	0.3208	10.2	834.9	837.3	2.4
36	107	71	35	80	0.3271	10.3	836.6	838.3	1.7
36	108	72	36	80	0.3333	10.5	838.0	841.4	3.4
36	109	73	37	80	0.3395	10.6	839.2	842.0	2.8
36	110	74	38	80	0.3455	10.8	840.1	844.8	4.7
36	111	75	39	80	0.3514	10.9	840.7	845.1	4.4
36	112	76	40	80	0.3571	11.1	841.1	847.5	6.4
36	113	77	41	80	0.3628	11.3	841.2	847.5	6.3
36	114	78	42	80	0.3684	11.4	841.1	849.5	8.5
36	115	79	43	80	0.3739	11.6	840.6	849.2	8.6
36	116	80	44	80	0.3793	11.8	839.9	851.0	11.0
36	117	81	45	80	0.3846	11.9	839.0	850.3	11.4
36	118	82	46	80	0.3898	12.1	837.8	851.8	14.0

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
41	78	37	-4	93	0.0513	8.7	632.6	627.9	-4.7
41	79	38	-3	93	0.0380	8.6	646.1	643.7	-2.4
41	80	39	-2	93	0.0250	8.6	659.3	657.0	-2.3
41	81	40	-1	93	0.0124	8.6	672.2	671.8	-0.3
41	82	41	0	93	0.0000	8.6	684.6	684.1	-0.5
41	83	42	1	93	0.0121	8.6	696.8	698.0	1.2
41	84	43	2	93	0.0238	8.6	708.6	709.5	0.9
41	85	44	3	93	0.0353	8.6	720.1	722.5	2.4
41	86	45	4	93	0.0465	8.7	731.3	733.1	1.8
41	87	46	5	93	0.0575	8.7	742.2	745.3	3.1
41	88	47	6	93	0.0682	8.8	752.7	755.1	2.3
41	89	48	7	93	0.0787	8.8	763.0	766.5	3.5
41	90	49	8	93	0.0889	8.9	773.0	775.6	2.5
41	91	50	9	93	0.0989	9.0	782.8	786.2	3.5
41	92	51	10	93	0.1087	9.0	792.2	794.6	2.4
41	93	52	11	93	0.1183	9.1	801.4	804.6	3.2
41	94	53	12	93	0.1277	9.2	810.2	812.3	2.1
41	95	54	13	93	0.1368	9.3	818.9	821.6	2.8
41	96	55	14	93	0.1458	9.4	827.2	828.8	1.5
41	97	56	15	93	0.1546	9.5	835.3	837.4	2.1
41	98	57	16	93	0.1633	9.6	843.2	844.0	0.8
41	99	58	17	93	0.1717	9.8	850.7	852.0	1.3
41	100	59	18	93	0.1800	9.9	858.0	858.0	0.0
41	101	60	19	93	0.1881	10.0	865.1	865.5	0.4
41	102	61	20	93	0.1961	10.1	871.9	871.0	-1.0
41	103	62	21	93	0.2039	10.3	878.5	877.9	-0.6
41	104	63	22	93	0.2115	10.4	884.8	882.9	-1.9
41	105	64	23	93	0.2191	10.5	890.8	889.3	-1.5
41	106	65	24	93	0.2264	10.7	896.7	893.8	-2.9
41	107	66	25	93	0.2337	10.8	902.2	899.7	-2.5
41	108	67	26	93	0.2407	11.0	907.5	903.7	-3.8
41	109	68	27	93	0.2477	11.1	912.6	909.2	-3.4
41	110	69	28	93	0.2546	11.3	917.5	912.8	-4.7
41	111	70	29	93	0.2613	11.4	922.1	917.9	-4.2
41	112	71	30	93	0.2679	11.6	926.4	921.0	-5.4
41	113	72	31	93	0.2743	11.7	930.5	925.6	-4.9
41	114	73	32	93	0.2807	11.9	934.4	928.4	-6.0
41	115	74	33	93	0.2870	12.1	938.1	932.6	-5.5
41	116	75	34	93	0.2931	12.2	941.5	935.0	-6.5
41	117	76	35	93	0.2992	12.4	944.6	938.8	-5.8
41	118	77	36	93	0.3051	12.6	947.6	940.8	-6.8
41	119	78	37	93	0.3109	12.7	950.3	944.3	-6.0
41	120	79	38	93	0.3167	12.9	952.7	945.9	-6.8
41	121	80	39	93	0.3223	13.1	955.0	949.1	-5.9
41	122	81	40	93	0.3279	13.3	957.0	950.4	-6.6
41	123	82	41	93	0.3333	13.4	958.7	953.2	-5.6
41	124	83	42	93	0.3387	13.6	960.3	954.1	-6.1
41	125	84	43	93	0.3440	13.8	961.6	956.6	-5.0
41	126	85	44	93	0.3492	14.0	962.7	957.3	-5.4
41	127	86	45	93	0.3543	14.2	963.5	959.4	-4.1
41	128	87	46	93	0.3594	14.3	964.1	959.8	-4.3
41	129	88	47	93	0.3643	14.5	964.5	961.6	-2.8
41	130	89	48	93	0.3692	14.7	964.6	961.7	-2.9
41	131	90	49	93	0.3741	14.9	964.6	963.3	-1.3
41	132	91	50	93	0.3788	15.1	964.2	963.1	-1.1

41	133	92	51	93	0.3835	15.3	963.7	964.4	0.7
41	134	93	52	93	0.3881	15.5	962.9	964.0	1.0
41	135	94	53	93	0.3926	15.7	962.0	965.0	3.0
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
46	88	42	-4	106	0.0455	10.8	704.3	703.1	-1.2
46	89	43	-3	106	0.0337	10.7	718.1	716.6	-1.4
46	90	44	-2	106	0.0222	10.7	731.5	732.4	0.9
46	91	45	-1	106	0.0110	10.7	744.5	745.0	0.5
46	92	46	0	106	0.0000	10.7	757.3	759.9	2.6
46	93	47	1	106	0.0108	10.7	769.8	771.7	1.9
46	94	48	2	106	0.0213	10.7	781.9	785.7	3.8
46	95	49	3	106	0.0316	10.7	793.7	796.8	3.0
46	96	50	4	106	0.0417	10.7	805.3	810.0	4.7
46	97	51	5	106	0.0516	10.8	816.6	820.3	3.7
46	98	52	6	106	0.0612	10.9	827.6	832.8	5.2
46	99	53	7	106	0.0707	10.9	838.3	842.4	4.1
46	100	54	8	106	0.0800	11.0	848.8	854.2	5.4
46	101	55	9	106	0.0891	11.1	859.0	863.2	4.1
46	102	56	10	106	0.0980	11.1	869.0	874.3	5.3
46	103	57	11	106	0.1068	11.2	878.7	882.6	3.9
46	104	58	12	106	0.1154	11.3	888.1	893.0	4.9
46	105	59	13	106	0.1238	11.4	897.3	900.8	3.4
46	106	60	14	106	0.1321	11.5	906.3	910.6	4.3
46	107	61	15	106	0.1402	11.7	915.0	917.8	2.8
46	108	62	16	106	0.1482	11.8	923.5	927.0	3.5
46	109	63	17	106	0.1560	11.9	931.7	933.6	1.9
46	110	64	18	106	0.1636	12.0	939.8	942.3	2.6
46	111	65	19	106	0.1712	12.1	947.5	948.4	0.9
46	112	66	20	106	0.1786	12.3	955.1	956.6	1.5
46	113	67	21	106	0.1858	12.4	962.4	962.2	-0.2
46	114	68	22	106	0.1930	12.6	969.5	969.9	0.4
46	115	69	23	106	0.2000	12.7	976.4	975.0	-1.4
46	116	70	24	106	0.2069	12.9	983.0	982.2	-0.8
46	117	71	25	106	0.2137	13.0	989.4	986.9	-2.6
46	118	72	26	106	0.2203	13.2	995.6	993.6	-2.0
46	119	73	27	106	0.2269	13.3	1001.6	997.8	-3.8
46	120	74	28	106	0.2333	13.5	1007.4	1004.1	-3.3
46	121	75	29	106	0.2397	13.6	1012.9	1007.9	-5.0
46	122	76	30	106	0.2459	13.8	1018.3	1013.8	-4.5
46	123	77	31	106	0.2520	14.0	1023.4	1017.2	-6.1
46	124	78	32	106	0.2581	14.2	1028.3	1022.7	-5.6
46	125	79	33	106	0.2640	14.3	1032.9	1025.7	-7.2
46	126	80	34	106	0.2698	14.5	1037.4	1030.8	-6.6
46	127	81	35	106	0.2756	14.7	1041.7	1033.5	-8.2
46	128	82	36	106	0.2813	14.9	1045.7	1038.1	-7.6
46	129	83	37	106	0.2868	15.0	1049.6	1040.5	-9.1
46	130	84	38	106	0.2923	15.2	1053.2	1044.8	-8.4
46	131	85	39	106	0.2977	15.4	1056.6	1046.8	-9.8
46	132	86	40	106	0.3030	15.6	1059.8	1050.8	-9.0
46	133	87	41	106	0.3083	15.8	1062.8	1052.4	-10.4
46	134	88	42	106	0.3134	16.0	1065.6	1056.1	-9.5
46	135	89	43	106	0.3185	16.2	1068.2	1057.4	-10.7
46	136	90	44	106	0.3235	16.4	1070.5	1060.8	-9.8
46	137	91	45	106	0.3285	16.6	1072.7	1061.8	-10.9
46	138	92	46	106	0.3333	16.8	1074.7	1064.8	-9.8
46	139	93	47	106	0.3381	17.0	1076.4	1065.6	-10.8

46	140	94	48	106	0.3429	17.2	1078.0	1068.3	-9.7
46	141	95	49	106	0.3475	17.4	1079.3	1068.8	-10.5
46	142	96	50	106	0.3521	17.6	1080.5	1071.2	-9.2
46	143	97	51	106	0.3566	17.8	1081.4	1071.5	-9.9
46	144	98	52	106	0.3611	18.0	1082.1	1073.6	-8.5
46	145	99	53	106	0.3655	18.2	1082.7	1073.6	-9.1
46	146	100	54	106	0.3699	18.4	1083.0	1075.5	-7.5
46	147	101	55	106	0.3742	18.6	1083.1	1075.2	-7.9
46	148	102	56	106	0.3784	18.8	1083.0	1076.8	-6.2
46	149	103	57	106	0.3826	19.1	1082.8	1076.3	-6.5
46	150	104	58	106	0.3867	19.3	1082.3	1077.7	-4.6
46	151	105	59	106	0.3907	19.5	1081.6	1076.9	-4.7
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
51	98	47	-4	119	0.0408	13.1	773.6	770.2	-3.4
51	99	48	-3	119	0.0303	13.0	787.5	786.1	-1.4
51	100	49	-2	119	0.0200	13.0	801.1	799.7	-1.4
51	101	50	-1	119	0.0099	13.0	814.4	814.8	0.4
51	102	51	0	119	0.0000	13.0	827.3	827.6	0.2
51	103	52	1	119	0.0097	13.0	840.0	841.9	1.9
51	104	53	2	119	0.0192	13.0	852.4	853.9	1.5
51	105	54	3	119	0.0286	13.0	864.6	867.5	2.9
51	106	55	4	119	0.0377	13.1	876.4	878.8	2.4
51	107	56	5	119	0.0467	13.1	888.0	891.7	3.7
51	108	57	6	119	0.0556	13.2	899.4	902.3	3.0
51	109	58	7	119	0.0642	13.2	910.5	914.5	4.1
51	110	59	8	119	0.0727	13.3	921.3	924.5	3.2
51	111	60	9	119	0.0811	13.4	931.9	936.1	4.2
51	112	61	10	119	0.0893	13.5	942.2	945.5	3.2
51	113	62	11	119	0.0974	13.6	952.4	956.4	4.0
51	114	63	12	119	0.1053	13.7	962.3	965.2	3.0
51	115	64	13	119	0.1130	13.8	971.9	975.6	3.6
51	116	65	14	119	0.1207	13.9	981.3	983.8	2.5
51	117	66	15	119	0.1282	14.0	990.6	993.6	3.0
51	118	67	16	119	0.1356	14.1	999.5	1001.3	1.8
51	119	68	17	119	0.1429	14.3	1008.3	1010.6	2.3
51	120	69	18	119	0.1500	14.4	1016.9	1017.8	0.9
51	121	70	19	119	0.1570	14.5	1025.2	1026.5	1.3
51	122	71	20	119	0.1639	14.7	1033.3	1033.2	-0.1
51	123	72	21	119	0.1707	14.8	1041.2	1041.5	0.2
51	124	73	22	119	0.1774	15.0	1048.9	1047.7	-1.2
51	125	74	23	119	0.1840	15.1	1056.4	1055.5	-0.9
51	126	75	24	119	0.1905	15.3	1063.7	1061.3	-2.4
51	127	76	25	119	0.1969	15.4	1070.8	1068.6	-2.2
51	128	77	26	119	0.2031	15.6	1077.6	1074.0	-3.7
51	129	78	27	119	0.2093	15.8	1084.3	1080.8	-3.5
51	130	79	28	119	0.2154	15.9	1090.8	1085.8	-5.0
51	131	80	29	119	0.2214	16.1	1097.1	1092.2	-4.8
51	132	81	30	119	0.2273	16.3	1103.1	1096.8	-6.3
51	133	82	31	119	0.2331	16.5	1109.0	1102.9	-6.2
51	134	83	32	119	0.2388	16.6	1114.7	1107.0	-7.7
51	135	84	33	119	0.2444	16.8	1120.2	1112.7	-7.5
51	136	85	34	119	0.2500	17.0	1125.5	1116.5	-8.9
51	137	86	35	119	0.2555	17.2	1130.6	1121.8	-8.8
51	138	87	36	119	0.2609	17.4	1135.5	1125.3	-10.2
51	139	88	37	119	0.2662	17.6	1140.2	1130.2	-10.0
51	140	89	38	119	0.2714	17.8	1144.7	1133.3	-11.4

51	141	90	39	119	0.2766	18.0	1149.0	1137.9	-11.1
51	142	91	40	119	0.2817	18.2	1153.1	1140.7	-12.4
51	143	92	41	119	0.2867	18.4	1157.1	1144.9	-12.1
51	144	93	42	119	0.2917	18.6	1160.8	1147.4	-13.4
51	145	94	43	119	0.2966	18.8	1164.4	1151.3	-13.0
51	146	95	44	119	0.3014	19.0	1167.8	1153.5	-14.2
51	147	96	45	119	0.3061	19.2	1170.9	1157.1	-13.8
51	148	97	46	119	0.3108	19.4	1173.9	1159.0	-14.9
51	149	98	47	119	0.3154	19.6	1176.8	1162.3	-14.4
51	150	99	48	119	0.3200	19.8	1179.4	1163.9	-15.4
51	151	100	49	119	0.3245	20.1	1181.8	1167.0	-14.9
51	152	101	50	119	0.3290	20.3	1184.1	1168.3	-15.8
51	153	102	51	119	0.3333	20.5	1186.1	1171.0	-15.1
51	154	103	52	119	0.3377	20.7	1188.0	1172.1	-15.9
51	155	104	53	119	0.3419	20.9	1189.7	1174.6	-15.2
51	156	105	54	119	0.3462	21.2	1191.2	1175.4	-15.9
51	157	106	55	119	0.3503	21.4	1192.6	1177.6	-15.0
51	158	107	56	119	0.3544	21.6	1193.7	1178.1	-15.6
51	159	108	57	119	0.3585	21.9	1194.7	1180.1	-14.6
51	160	109	58	119	0.3625	22.1	1195.4	1180.4	-15.1
51	161	110	59	119	0.3665	22.3	1196.0	1182.1	-13.9
51	162	111	60	119	0.3704	22.6	1196.5	1182.2	-14.3
51	163	112	61	119	0.3742	22.8	1196.7	1183.7	-13.0
51	164	113	62	119	0.3781	23.0	1196.8	1183.5	-13.2
51	165	114	63	119	0.3818	23.3	1196.6	1184.8	-11.9
51	166	115	64	119	0.3855	23.5	1196.3	1184.4	-11.9
51	167	116	65	119	0.3892	23.7	1195.8	1185.4	-10.4
51	168	117	66	119	0.3929	24.0	1195.2	1184.8	-10.3

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
56	108	52	-4	132	0.0370	15.7	840.5	838.5	-2.0
56	109	53	-3	132	0.0275	15.6	854.5	852.3	-2.3
56	110	54	-2	132	0.0182	15.6	868.2	868.2	-0.1
56	111	55	-1	132	0.0090	15.6	881.7	881.2	-0.5
56	112	56	0	132	0.0000	15.6	894.8	896.4	1.5
56	113	57	1	132	0.0089	15.6	907.7	908.7	1.0
56	114	58	2	132	0.0175	15.6	920.3	923.1	2.8
56	115	59	3	132	0.0261	15.6	932.7	934.8	2.1
56	116	60	4	132	0.0345	15.6	944.8	948.5	3.8
56	117	61	5	132	0.0427	15.7	956.6	959.6	3.0
56	118	62	6	132	0.0509	15.8	968.2	972.7	4.4
56	119	63	7	132	0.0588	15.8	979.6	983.1	3.5
56	120	64	8	132	0.0667	15.9	990.7	995.6	4.8
56	121	65	9	132	0.0744	16.0	1001.6	1005.4	3.8
56	122	66	10	132	0.0820	16.1	1012.3	1017.3	5.0
56	123	67	11	132	0.0894	16.2	1022.8	1026.6	3.8
56	124	68	12	132	0.0968	16.3	1033.0	1037.9	4.9
56	125	69	13	132	0.1040	16.4	1043.0	1046.6	3.6
56	126	70	14	132	0.1111	16.5	1052.8	1057.4	4.5
56	127	71	15	132	0.1181	16.6	1062.4	1065.6	3.1
56	128	72	16	132	0.1250	16.7	1071.8	1075.8	4.0
56	129	73	17	132	0.1318	16.9	1081.0	1083.5	2.5
56	130	74	18	132	0.1385	17.0	1090.0	1093.3	3.3
56	131	75	19	132	0.1450	17.1	1098.8	1100.5	1.8
56	132	76	20	132	0.1515	17.3	1107.3	1109.8	2.4
56	133	77	21	132	0.1579	17.4	1115.7	1116.5	0.8
56	134	78	22	132	0.1642	17.6	1123.9	1125.3	1.4

56	135	79	23	132	0.1704	17.7	1131.9	1131.7	-0.2
56	136	80	24	132	0.1765	17.9	1139.7	1140.0	0.3
56	137	81	25	132	0.1825	18.1	1147.3	1145.9	-1.4
56	138	82	26	132	0.1884	18.2	1154.7	1153.8	-0.9
56	139	83	27	132	0.1943	18.4	1161.9	1159.3	-2.6
56	140	84	28	132	0.2000	18.6	1168.9	1166.8	-2.1
56	141	85	29	132	0.2057	18.8	1175.8	1171.9	-3.9
56	142	86	30	132	0.2113	19.0	1182.5	1179.0	-3.4
56	143	87	31	132	0.2168	19.1	1188.9	1183.8	-5.2
56	144	88	32	132	0.2222	19.3	1195.2	1190.5	-4.8
56	145	89	33	132	0.2276	19.5	1201.3	1194.9	-6.5
56	146	90	34	132	0.2329	19.7	1207.3	1201.2	-6.1
56	147	91	35	132	0.2381	19.9	1213.0	1205.2	-7.8
56	148	92	36	132	0.2432	20.1	1218.6	1211.2	-7.4
56	149	93	37	132	0.2483	20.3	1224.0	1214.9	-9.1
56	150	94	38	132	0.2533	20.5	1229.2	1220.5	-8.7
56	151	95	39	132	0.2583	20.7	1234.2	1223.8	-10.4
56	152	96	40	132	0.2632	21.0	1239.1	1229.1	-10.0
56	153	97	41	132	0.2680	21.2	1243.8	1232.2	-11.6
56	154	98	42	132	0.2727	21.4	1248.3	1237.1	-11.2
56	155	99	43	132	0.2774	21.6	1252.6	1239.9	-12.8
56	156	100	44	132	0.2821	21.8	1256.8	1244.5	-12.3
56	157	101	45	132	0.2866	22.0	1260.8	1247.0	-13.8
56	158	102	46	132	0.2911	22.3	1264.6	1251.3	-13.3
56	159	103	47	132	0.2956	22.5	1268.2	1253.5	-14.8
56	160	104	48	132	0.3000	22.7	1271.7	1257.5	-14.2
56	161	105	49	132	0.3044	23.0	1275.0	1259.4	-15.6
56	162	106	50	132	0.3086	23.2	1278.1	1263.2	-15.0
56	163	107	51	132	0.3129	23.4	1281.1	1264.8	-16.3
56	164	108	52	132	0.3171	23.7	1283.9	1268.3	-15.6
56	165	109	53	132	0.3212	23.9	1286.5	1269.6	-16.9
56	166	110	54	132	0.3253	24.1	1288.9	1272.8	-16.1
56	167	111	55	132	0.3293	24.4	1291.2	1274.0	-17.3
56	168	112	56	132	0.3333	24.6	1293.3	1276.9	-16.4
56	169	113	57	132	0.3373	24.9	1295.3	1277.8	-17.5
56	170	114	58	132	0.3412	25.1	1297.0	1280.5	-16.6
56	171	115	59	132	0.3450	25.4	1298.6	1281.1	-17.5
56	172	116	60	132	0.3488	25.6	1300.1	1283.6	-16.5
56	173	117	61	132	0.3526	25.9	1301.4	1284.0	-17.4
56	174	118	62	132	0.3563	26.1	1302.5	1286.2	-16.2
56	175	119	63	132	0.3600	26.4	1303.4	1286.4	-17.0
56	176	120	64	132	0.3636	26.6	1304.2	1288.4	-15.8
56	177	121	65	132	0.3672	26.9	1304.8	1288.4	-16.4
56	178	122	66	132	0.3708	27.1	1305.2	1290.1	-15.1
56	179	123	67	132	0.3743	27.4	1305.5	1289.9	-15.6
56	180	124	68	132	0.3778	27.6	1305.6	1291.5	-14.1
56	181	125	69	132	0.3812	27.9	1305.6	1291.0	-14.5
56	182	126	70	132	0.3846	28.2	1305.3	1292.4	-13.0
56	183	127	71	132	0.3880	28.4	1305.0	1291.7	-13.2
56	184	128	72	132	0.3913	28.7	1304.4	1292.9	-11.5

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
61	118	57	-4	145	0.0339	18.5	905.0	899.0	-6.0
61	119	58	-3	145	0.0252	18.4	919.1	915.1	-4.0
61	120	59	-2	145	0.0167	18.4	933.0	928.9	-4.0
61	121	60	-1	145	0.0083	18.4	946.5	944.3	-2.3
61	122	61	0	145	0.0000	18.4	959.8	957.4	-2.4

61	123	62	1	145	0.0081	18.4	972.9	972.1	-0.8
61	124	63	2	145	0.0161	18.4	985.6	984.6	-1.0
61	125	64	3	145	0.0240	18.4	998.2	998.6	0.4
61	126	65	4	145	0.0318	18.5	1010.5	1010.5	0.0
61	127	66	5	145	0.0394	18.5	1022.5	1023.9	1.4
61	128	67	6	145	0.0469	18.6	1034.4	1035.2	0.9
61	129	68	7	145	0.0543	18.6	1046.0	1048.0	2.1
61	130	69	8	145	0.0615	18.7	1057.3	1058.8	1.4
61	131	70	9	145	0.0687	18.8	1068.5	1071.0	2.5
61	132	71	10	145	0.0758	18.9	1079.4	1081.2	1.7
61	133	72	11	145	0.0827	19.0	1090.2	1092.9	2.7
61	134	73	12	145	0.0896	19.1	1100.7	1102.5	1.8
61	135	74	13	145	0.0963	19.2	1111.0	1113.7	2.7
61	136	75	14	145	0.1029	19.3	1121.1	1122.8	1.7
61	137	76	15	145	0.1095	19.4	1131.0	1133.5	2.4
61	138	77	16	145	0.1159	19.6	1140.8	1142.1	1.4
61	139	78	17	145	0.1223	19.7	1150.3	1152.3	2.0
61	140	79	18	145	0.1286	19.8	1159.6	1160.5	0.9
61	141	80	19	145	0.1348	20.0	1168.7	1170.2	1.4
61	142	81	20	145	0.1409	20.1	1177.7	1177.9	0.3
61	143	82	21	145	0.1469	20.3	1186.4	1187.2	0.7
61	144	83	22	145	0.1528	20.4	1195.0	1194.5	-0.5
61	145	84	23	145	0.1586	20.6	1203.4	1203.3	-0.1
61	146	85	24	145	0.1644	20.8	1211.6	1210.2	-1.4
61	147	86	25	145	0.1701	20.9	1219.6	1218.5	-1.1
61	148	87	26	145	0.1757	21.1	1227.5	1225.0	-2.4
61	149	88	27	145	0.1812	21.3	1235.1	1233.0	-2.1
61	150	89	28	145	0.1867	21.5	1242.6	1239.1	-3.5
61	151	90	29	145	0.1921	21.7	1249.9	1246.7	-3.3
61	152	91	30	145	0.1974	21.9	1257.1	1252.4	-4.7
61	153	92	31	145	0.2026	22.1	1264.0	1259.6	-4.5
61	154	93	32	145	0.2078	22.3	1270.8	1265.0	-5.9
61	155	94	33	145	0.2129	22.5	1277.5	1271.8	-5.7
61	156	95	34	145	0.2180	22.7	1283.9	1276.8	-7.1
61	157	96	35	145	0.2229	22.9	1290.2	1283.2	-7.0
61	158	97	36	145	0.2279	23.1	1296.3	1287.9	-8.4
61	159	98	37	145	0.2327	23.3	1302.3	1294.0	-8.2
61	160	99	38	145	0.2375	23.5	1308.0	1298.4	-9.6
61	161	100	39	145	0.2422	23.7	1313.7	1304.2	-9.5
61	162	101	40	145	0.2469	24.0	1319.1	1308.2	-10.9
61	163	102	41	145	0.2515	24.2	1324.4	1313.7	-10.7
61	164	103	42	145	0.2561	24.4	1329.5	1317.4	-12.1
61	165	104	43	145	0.2606	24.6	1334.5	1322.5	-11.9
61	166	105	44	145	0.2651	24.9	1339.3	1326.0	-13.3
61	167	106	45	145	0.2695	25.1	1343.9	1330.8	-13.1
61	168	107	46	145	0.2738	25.3	1348.4	1334.0	-14.4
61	169	108	47	145	0.2781	25.6	1352.7	1338.5	-14.2
61	170	109	48	145	0.2824	25.8	1356.8	1341.4	-15.4
61	171	110	49	145	0.2866	26.1	1360.8	1345.6	-15.2
61	172	111	50	145	0.2907	26.3	1364.6	1348.2	-16.4
61	173	112	51	145	0.2948	26.6	1368.3	1352.2	-16.1
61	174	113	52	145	0.2989	26.8	1371.8	1354.6	-17.3
61	175	114	53	145	0.3029	27.0	1375.2	1358.3	-16.9
61	176	115	54	145	0.3068	27.3	1378.4	1360.4	-18.0
61	177	116	55	145	0.3107	27.6	1381.4	1363.8	-17.6
61	178	117	56	145	0.3146	27.8	1384.3	1365.6	-18.7
61	179	118	57	145	0.3184	28.1	1387.0	1368.9	-18.2
61	180	119	58	145	0.3222	28.3	1389.6	1370.4	-19.2
61	181	120	59	145	0.3260	28.6	1392.0	1373.4	-18.6

61	182	121	60	145	0.3297	28.9	1394.3	1374.8	-19.5
61	183	122	61	145	0.3333	29.1	1396.4	1377.5	-18.9
61	184	123	62	145	0.3370	29.4	1398.3	1378.6	-19.7
61	185	124	63	145	0.3405	29.6	1400.1	1381.1	-19.0
61	186	125	64	145	0.3441	29.9	1401.8	1382.0	-19.7
61	187	126	65	145	0.3476	30.2	1403.2	1384.3	-19.0
61	188	127	66	145	0.3511	30.5	1404.6	1385.0	-19.6
61	189	128	67	145	0.3545	30.7	1405.8	1387.0	-18.7
61	190	129	68	145	0.3579	31.0	1406.8	1387.5	-19.3
61	191	130	69	145	0.3613	31.3	1407.7	1389.4	-18.3
61	192	131	70	145	0.3646	31.6	1408.4	1389.6	-18.8
61	193	132	71	145	0.3679	31.8	1409.0	1391.3	-17.7
61	194	133	72	145	0.3711	32.1	1409.4	1391.4	-18.0
61	195	134	73	145	0.3744	32.4	1409.7	1392.8	-16.9
61	196	135	74	145	0.3776	32.7	1409.8	1392.7	-17.1
61	197	136	75	145	0.3807	33.0	1409.7	1393.9	-15.8
61	198	137	76	145	0.3838	33.2	1409.6	1393.6	-15.9
61	199	138	77	145	0.3869	33.5	1409.2	1394.7	-14.6
61	200	139	78	145	0.3900	33.8	1408.7	1394.2	-14.6
61	201	140	79	145	0.3930	34.1	1408.1	1395.0	-13.1

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
66	128	62	-4	160	0.0313	21.5	960.0	960.6	0.6
66	129	63	-3	160	0.0233	21.5	974.4	974.6	0.2
66	130	64	-2	160	0.0154	21.4	988.5	990.7	2.1
66	131	65	-1	160	0.0076	21.4	1002.4	1004.0	1.6
66	132	66	0	160	0.0000	21.4	1016.1	1019.4	3.4
66	133	67	1	160	0.0075	21.4	1029.4	1032.2	2.8
66	134	68	2	160	0.0149	21.4	1042.6	1047.0	4.4
66	135	69	3	160	0.0222	21.5	1055.5	1059.1	3.6
66	136	70	4	160	0.0294	21.5	1068.2	1073.3	5.1
66	137	71	5	160	0.0365	21.6	1080.6	1084.9	4.3
66	138	72	6	160	0.0435	21.6	1092.9	1098.5	5.7
66	139	73	7	160	0.0504	21.7	1104.9	1109.6	4.7
66	140	74	8	160	0.0571	21.8	1116.7	1122.6	5.9
66	141	75	9	160	0.0638	21.8	1128.3	1133.1	4.9
66	142	76	10	160	0.0704	21.9	1139.7	1145.6	6.0
66	143	77	11	160	0.0769	22.0	1150.8	1155.7	4.8
66	144	78	12	160	0.0833	22.1	1161.8	1167.7	5.8
66	145	79	13	160	0.0897	22.2	1172.6	1177.2	4.6
66	146	80	14	160	0.0959	22.4	1183.2	1188.7	5.5
66	147	81	15	160	0.1020	22.5	1193.6	1197.8	4.1
66	148	82	16	160	0.1081	22.6	1203.9	1208.8	4.9
66	149	83	17	160	0.1141	22.8	1213.9	1217.4	3.5
66	150	84	18	160	0.1200	22.9	1223.7	1227.9	4.2
66	151	85	19	160	0.1258	23.1	1233.4	1236.1	2.7
66	152	86	20	160	0.1316	23.2	1242.9	1246.2	3.3
66	153	87	21	160	0.1373	23.4	1252.2	1254.0	1.8
66	154	88	22	160	0.1429	23.5	1261.4	1263.7	2.3
66	155	89	23	160	0.1484	23.7	1270.3	1271.0	0.7
66	156	90	24	160	0.1539	23.9	1279.1	1280.3	1.1
66	157	91	25	160	0.1592	24.1	1287.8	1287.2	-0.5
66	158	92	26	160	0.1646	24.2	1296.2	1296.1	-0.2
66	159	93	27	160	0.1698	24.4	1304.5	1302.6	-1.9
66	160	94	28	160	0.1750	24.6	1312.6	1311.1	-1.5
66	161	95	29	160	0.1801	24.8	1320.6	1317.3	-3.3
66	162	96	30	160	0.1852	25.0	1328.4	1325.4	-3.0

66	163	97	31	160	0.1902	25.2	1336.0	1331.2	-4.8
66	164	98	32	160	0.1951	25.4	1343.4	1338.9	-4.5
66	165	99	33	160	0.2000	25.6	1350.8	1344.4	-6.3
66	166	100	34	160	0.2048	25.8	1357.9	1351.8	-6.1
66	167	101	35	160	0.2096	26.1	1364.9	1357.0	-7.9
66	168	102	36	160	0.2143	26.3	1371.7	1364.0	-7.8
66	169	103	37	160	0.2189	26.5	1378.4	1368.8	-9.6
66	170	104	38	160	0.2235	26.7	1384.9	1375.5	-9.4
66	171	105	39	160	0.2281	27.0	1391.3	1380.0	-11.2
66	172	106	40	160	0.2326	27.2	1397.5	1386.4	-11.1
66	173	107	41	160	0.2370	27.4	1403.5	1390.6	-12.9
66	174	108	42	160	0.2414	27.7	1409.4	1396.6	-12.8
66	175	109	43	160	0.2457	27.9	1415.2	1400.6	-14.6
66	176	110	44	160	0.2500	28.1	1420.7	1406.3	-14.4
66	177	111	45	160	0.2542	28.4	1426.2	1409.9	-16.2
66	178	112	46	160	0.2584	28.6	1431.5	1415.4	-16.1
66	179	113	47	160	0.2626	28.9	1436.6	1418.7	-17.9
66	180	114	48	160	0.2667	29.1	1441.6	1423.9	-17.7
66	181	115	49	160	0.2707	29.4	1446.5	1427.0	-19.5
66	182	116	50	160	0.2747	29.6	1451.1	1431.9	-19.3
66	183	117	51	160	0.2787	29.9	1455.7	1434.7	-21.0
66	184	118	52	160	0.2826	30.2	1460.1	1439.3	-20.8
66	185	119	53	160	0.2865	30.4	1464.3	1441.9	-22.5
66	186	120	54	160	0.2903	30.7	1468.5	1446.2	-22.2
66	187	121	55	160	0.2941	30.9	1472.4	1448.6	-23.9
66	188	122	56	160	0.2979	31.2	1476.2	1452.7	-23.6
66	189	123	57	160	0.3016	31.5	1479.9	1454.7	-25.2
66	190	124	58	160	0.3053	31.8	1483.4	1458.6	-24.8
66	191	125	59	160	0.3089	32.0	1486.8	1460.4	-26.4
66	192	126	60	160	0.3125	32.3	1490.0	1464.0	-26.0
66	193	127	61	160	0.3161	32.6	1493.1	1465.7	-27.5
66	194	128	62	160	0.3196	32.9	1496.1	1469.0	-27.1
66	195	129	63	160	0.3231	33.1	1498.9	1470.4	-28.5
66	196	130	64	160	0.3265	33.4	1501.6	1473.6	-28.0
66	197	131	65	160	0.3300	33.7	1504.1	1474.7	-29.3
66	198	132	66	160	0.3333	34.0	1506.5	1477.7	-28.8
66	199	133	67	160	0.3367	34.3	1508.7	1478.6	-30.1
66	200	134	68	160	0.3400	34.6	1510.8	1481.3	-29.5
66	201	135	69	160	0.3433	34.9	1512.8	1482.1	-30.7
66	202	136	70	160	0.3465	35.2	1514.6	1484.6	-30.0
66	203	137	71	160	0.3498	35.4	1516.2	1485.1	-31.1
66	204	138	72	160	0.3529	35.7	1517.8	1487.4	-30.4
66	205	139	73	160	0.3561	36.0	1519.2	1487.8	-31.4
66	206	140	74	160	0.3592	36.3	1520.4	1489.9	-30.6
66	207	141	75	160	0.3623	36.6	1521.5	1490.1	-31.5
66	208	142	76	160	0.3654	36.9	1522.5	1491.9	-30.6
66	209	143	77	160	0.3684	37.2	1523.4	1491.9	-31.4
66	210	144	78	160	0.3714	37.5	1524.1	1493.6	-30.4
66	211	145	79	160	0.3744	37.8	1524.6	1493.5	-31.2
66	212	146	80	160	0.3774	38.1	1525.0	1495.0	-30.1
66	213	147	81	160	0.3803	38.5	1525.3	1494.6	-30.7
66	214	148	82	160	0.3832	38.8	1525.5	1495.9	-29.6
66	215	149	83	160	0.3861	39.1	1525.5	1495.4	-30.1
66	216	150	84	160	0.3889	39.4	1525.4	1496.5	-28.8
66	217	151	85	160	0.3917	39.7	1525.1	1495.8	-29.3
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)

71	138	67	-4	173	0.0290	24.8	1019.6	1014.7	-4.9
71	139	68	-3	173	0.0216	24.8	1034.1	1030.9	-3.2
71	140	69	-2	173	0.0143	24.7	1048.3	1044.9	-3.4
71	141	70	-1	173	0.0071	24.7	1062.3	1060.5	-1.7
71	142	71	0	173	0.0000	24.7	1076.0	1074.0	-2.0
71	143	72	1	173	0.0070	24.7	1089.4	1089.0	-0.5
71	144	73	2	173	0.0139	24.7	1102.7	1101.9	-0.8
71	145	74	3	173	0.0207	24.8	1115.7	1116.3	0.6
71	146	75	4	173	0.0274	24.8	1128.5	1128.6	0.1
71	147	76	5	173	0.0340	24.8	1141.1	1142.5	1.4
71	148	77	6	173	0.0405	24.9	1153.5	1154.3	0.8
71	149	78	7	173	0.0470	25.0	1165.6	1167.6	2.0
71	150	79	8	173	0.0533	25.0	1177.6	1178.9	1.4
71	151	80	9	173	0.0596	25.1	1189.3	1191.7	2.4
71	152	81	10	173	0.0658	25.2	1200.9	1202.6	1.7
71	153	82	11	173	0.0719	25.3	1212.2	1214.8	2.6
71	154	83	12	173	0.0779	25.4	1223.4	1225.2	1.8
71	155	84	13	173	0.0839	25.5	1234.4	1237.0	2.6
71	156	85	14	173	0.0897	25.7	1245.2	1246.9	1.7
71	157	86	15	173	0.0955	25.8	1255.8	1258.3	2.4
71	158	87	16	173	0.1013	25.9	1266.2	1267.7	1.5
71	159	88	17	173	0.1069	26.1	1276.5	1278.6	2.1
71	160	89	18	173	0.1125	26.2	1286.6	1287.6	1.0
71	161	90	19	173	0.1180	26.4	1296.5	1298.1	1.6
71	162	91	20	173	0.1235	26.5	1306.2	1306.7	0.5
71	163	92	21	173	0.1288	26.7	1315.8	1316.7	1.0
71	164	93	22	173	0.1342	26.9	1325.1	1324.9	-0.2
71	165	94	23	173	0.1394	27.0	1334.4	1334.6	0.2
71	166	95	24	173	0.1446	27.2	1343.4	1342.4	-1.1
71	167	96	25	173	0.1497	27.4	1352.3	1351.6	-0.7
71	168	97	26	173	0.1548	27.6	1361.1	1359.0	-2.0
71	169	98	27	173	0.1598	27.8	1369.6	1367.9	-1.7
71	170	99	28	173	0.1647	28.0	1378.0	1375.0	-3.1
71	171	100	29	173	0.1696	28.2	1386.3	1383.4	-2.8
71	172	101	30	173	0.1744	28.4	1394.4	1390.1	-4.2
71	173	102	31	173	0.1792	28.6	1402.3	1398.3	-4.0
71	174	103	32	173	0.1839	28.8	1410.1	1404.6	-5.5
71	175	104	33	173	0.1886	29.0	1417.7	1412.4	-5.3
71	176	105	34	173	0.1932	29.2	1425.2	1418.5	-6.8
71	177	106	35	173	0.1977	29.5	1432.5	1425.9	-6.6
71	178	107	36	173	0.2023	29.7	1439.7	1431.6	-8.1
71	179	108	37	173	0.2067	29.9	1446.7	1438.7	-8.0
71	180	109	38	173	0.2111	30.2	1453.6	1444.1	-9.5
71	181	110	39	173	0.2155	30.4	1460.3	1450.9	-9.4
71	182	111	40	173	0.2198	30.6	1466.9	1456.0	-10.9
71	183	112	41	173	0.2240	30.9	1473.3	1462.5	-10.9
71	184	113	42	173	0.2283	31.1	1479.6	1467.3	-12.4
71	185	114	43	173	0.2324	31.4	1485.8	1473.4	-12.3
71	186	115	44	173	0.2366	31.6	1491.8	1477.9	-13.8
71	187	116	45	173	0.2406	31.9	1497.6	1483.8	-13.8
71	188	117	46	173	0.2447	32.1	1503.3	1488.1	-15.3
71	189	118	47	173	0.2487	32.4	1508.9	1493.7	-15.2
71	190	119	48	173	0.2526	32.7	1514.3	1497.6	-16.7
71	191	120	49	173	0.2566	32.9	1519.6	1502.9	-16.7
71	192	121	50	173	0.2604	33.2	1524.7	1506.6	-18.1
71	193	122	51	173	0.2643	33.5	1529.8	1511.7	-18.1
71	194	123	52	173	0.2680	33.7	1534.6	1515.1	-19.5
71	195	124	53	173	0.2718	34.0	1539.3	1519.9	-19.4
71	196	125	54	173	0.2755	34.3	1543.9	1523.1	-20.8

71	197	126	55	173	0.2792	34.6	1548.4	1527.6	-20.8
71	198	127	56	173	0.2828	34.8	1552.7	1530.6	-22.1
71	199	128	57	173	0.2864	35.1	1556.9	1534.9	-22.0
71	200	129	58	173	0.2900	35.4	1560.9	1537.5	-23.3
71	201	130	59	173	0.2935	35.7	1564.8	1541.6	-23.2
71	202	131	60	173	0.2970	36.0	1568.5	1544.1	-24.5
71	203	132	61	173	0.3005	36.3	1572.2	1547.9	-24.3
71	204	133	62	173	0.3039	36.6	1575.7	1550.1	-25.6
71	205	134	63	173	0.3073	36.9	1579.0	1553.7	-25.3
71	206	135	64	173	0.3107	37.1	1582.2	1555.7	-26.5
71	207	136	65	173	0.3140	37.4	1585.3	1559.1	-26.2
71	208	137	66	173	0.3173	37.7	1588.3	1560.9	-27.4
71	209	138	67	173	0.3206	38.0	1591.1	1564.0	-27.1
71	210	139	68	173	0.3238	38.3	1593.8	1565.6	-28.2
71	211	140	69	173	0.3270	38.7	1596.3	1568.6	-27.8
71	212	141	70	173	0.3302	39.0	1598.8	1570.0	-28.8
71	213	142	71	173	0.3333	39.3	1601.1	1572.7	-28.4
71	214	143	72	173	0.3365	39.6	1603.2	1573.9	-29.3
71	215	144	73	173	0.3395	39.9	1605.2	1576.4	-28.8
71	216	145	74	173	0.3426	40.2	1607.1	1577.4	-29.7
71	217	146	75	173	0.3456	40.5	1608.9	1579.7	-29.2
71	218	147	76	173	0.3486	40.8	1610.5	1580.5	-30.0
71	219	148	77	173	0.3516	41.1	1612.0	1582.7	-29.3
71	220	149	78	173	0.3546	41.5	1613.4	1583.3	-30.1
71	221	150	79	173	0.3575	41.8	1614.6	1585.2	-29.4
71	222	151	80	173	0.3604	42.1	1615.7	1585.7	-30.0
71	223	152	81	173	0.3632	42.4	1616.7	1587.5	-29.3
71	224	153	82	173	0.3661	42.7	1617.6	1587.7	-29.8
71	225	154	83	173	0.3689	43.1	1618.3	1589.3	-29.0
71	226	155	84	173	0.3717	43.4	1618.9	1589.4	-29.5
71	227	156	85	173	0.3745	43.7	1619.3	1590.8	-28.5
71	228	157	86	173	0.3772	44.0	1619.7	1590.7	-28.9
71	229	158	87	173	0.3799	44.4	1619.9	1592.0	-27.9
71	230	159	88	173	0.3826	44.7	1620.0	1591.7	-28.2
71	231	160	89	173	0.3853	45.0	1619.9	1592.8	-27.1
71	232	161	90	173	0.3879	45.4	1619.7	1592.4	-27.3
71	233	162	91	173	0.3906	45.7	1619.4	1593.3	-26.1
71	234	163	92	173	0.3932	46.0	1619.0	1592.8	-26.2

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
76	148	72	-4	188	0.0270	28.3	1069.3	1069.7	0.3
76	149	73	-3	188	0.0201	28.3	1084.0	1083.9	-0.1
76	150	74	-2	188	0.0133	28.3	1098.4	1100.1	1.7
76	151	75	-1	188	0.0066	28.2	1112.6	1113.8	1.1
76	152	76	0	188	0.0000	28.2	1126.6	1129.4	2.8
76	153	77	1	188	0.0065	28.2	1140.3	1142.5	2.2
76	154	78	2	188	0.0130	28.3	1153.9	1157.6	3.7
76	155	79	3	188	0.0194	28.3	1167.2	1170.2	3.0
76	156	80	4	188	0.0256	28.3	1180.2	1184.7	4.5
76	157	81	5	188	0.0319	28.4	1193.1	1196.8	3.7
76	158	82	6	188	0.0380	28.4	1205.8	1210.8	5.0
76	159	83	7	188	0.0440	28.5	1218.2	1222.4	4.1
76	160	84	8	188	0.0500	28.6	1230.5	1235.9	5.4
76	161	85	9	188	0.0559	28.7	1242.6	1247.0	4.4
76	162	86	10	188	0.0617	28.8	1254.5	1260.0	5.6
76	163	87	11	188	0.0675	28.9	1266.2	1270.7	4.5
76	164	88	12	188	0.0732	29.0	1277.7	1283.2	5.6

76	165	89	13	188	0.0788	29.1	1289.0	1293.5	4.4
76	166	90	14	188	0.0843	29.2	1300.2	1305.5	5.4
76	167	91	15	188	0.0898	29.3	1311.2	1315.3	4.2
76	168	92	16	188	0.0952	29.5	1322.0	1327.0	5.0
76	169	93	17	188	0.1006	29.6	1332.6	1336.3	3.8
76	170	94	18	188	0.1059	29.8	1343.1	1347.6	4.5
76	171	95	19	188	0.1111	29.9	1353.3	1356.5	3.2
76	172	96	20	188	0.1163	30.1	1363.5	1367.3	3.9
76	173	97	21	188	0.1214	30.3	1373.4	1375.9	2.5
76	174	98	22	188	0.1264	30.4	1383.2	1386.3	3.1
76	175	99	23	188	0.1314	30.6	1392.9	1394.5	1.6
76	176	100	24	188	0.1364	30.8	1402.4	1404.5	2.2
76	177	101	25	188	0.1412	31.0	1411.7	1412.3	0.7
76	178	102	26	188	0.1461	31.2	1420.9	1422.0	1.1
76	179	103	27	188	0.1508	31.4	1429.9	1429.4	-0.4
76	180	104	28	188	0.1556	31.6	1438.7	1438.7	0.0
76	181	105	29	188	0.1602	31.8	1447.5	1445.8	-1.6
76	182	106	30	188	0.1648	32.0	1456.0	1454.8	-1.3
76	183	107	31	188	0.1694	32.2	1464.4	1461.5	-2.9
76	184	108	32	188	0.1739	32.4	1472.7	1470.1	-2.6
76	185	109	33	188	0.1784	32.6	1480.8	1476.5	-4.3
76	186	110	34	188	0.1828	32.9	1488.8	1484.8	-4.0
76	187	111	35	188	0.1872	33.1	1496.6	1490.9	-5.7
76	188	112	36	188	0.1915	33.3	1504.3	1498.8	-5.5
76	189	113	37	188	0.1958	33.6	1511.8	1504.6	-7.2
76	190	114	38	188	0.2000	33.8	1519.2	1512.2	-7.0
76	191	115	39	188	0.2042	34.1	1526.5	1517.7	-8.8
76	192	116	40	188	0.2083	34.3	1533.6	1525.0	-8.6
76	193	117	41	188	0.2124	34.6	1540.6	1530.2	-10.4
76	194	118	42	188	0.2165	34.8	1547.4	1537.2	-10.2
76	195	119	43	188	0.2205	35.1	1554.1	1542.1	-12.0
76	196	120	44	188	0.2245	35.3	1560.7	1548.8	-11.9
76	197	121	45	188	0.2284	35.6	1567.1	1553.5	-13.6
76	198	122	46	188	0.2323	35.9	1573.4	1559.9	-13.5
76	199	123	47	188	0.2362	36.1	1579.6	1564.3	-15.3
76	200	124	48	188	0.2400	36.4	1585.6	1570.4	-15.2
76	201	125	49	188	0.2438	36.7	1591.5	1574.5	-17.0
76	202	126	50	188	0.2475	37.0	1597.2	1580.4	-16.9
76	203	127	51	188	0.2512	37.2	1602.8	1584.2	-18.6
76	204	128	52	188	0.2549	37.5	1608.3	1589.8	-18.5
76	205	129	53	188	0.2585	37.8	1613.7	1593.5	-20.2
76	206	130	54	188	0.2621	38.1	1618.9	1598.8	-20.1
76	207	131	55	188	0.2657	38.4	1624.0	1602.2	-21.9
76	208	132	56	188	0.2692	38.7	1629.0	1607.3	-21.7
76	209	133	57	188	0.2727	39.0	1633.8	1610.4	-23.4
76	210	134	58	188	0.2762	39.3	1638.5	1615.2	-23.3
76	211	135	59	188	0.2796	39.6	1643.1	1618.1	-25.0
76	212	136	60	188	0.2830	39.9	1647.5	1622.7	-24.8
76	213	137	61	188	0.2864	40.2	1651.9	1625.4	-26.5
76	214	138	62	188	0.2897	40.5	1656.1	1629.8	-26.3
76	215	139	63	188	0.2930	40.8	1660.1	1632.2	-27.9
76	216	140	64	188	0.2963	41.1	1664.1	1636.4	-27.7
76	217	141	65	188	0.2995	41.4	1667.9	1638.6	-29.2
76	218	142	66	188	0.3028	41.7	1671.6	1642.6	-29.0
76	219	143	67	188	0.3059	42.0	1675.1	1644.6	-30.5
76	220	144	68	188	0.3091	42.3	1678.6	1648.3	-30.2
76	221	145	69	188	0.3122	42.6	1681.9	1650.1	-31.7
76	222	146	70	188	0.3153	43.0	1685.0	1653.6	-31.4
76	223	147	71	188	0.3184	43.3	1688.1	1655.3	-32.8

76	224	148	72	188	0.3214	43.6	1691.0	1658.6	-32.5
76	225	149	73	188	0.3244	43.9	1693.9	1660.0	-33.9
76	226	150	74	188	0.3274	44.3	1696.5	1663.1	-33.5
76	227	151	75	188	0.3304	44.6	1699.1	1664.3	-34.8
76	228	152	76	188	0.3333	44.9	1701.5	1667.2	-34.3
76	229	153	77	188	0.3363	45.2	1703.9	1668.3	-35.6
76	230	154	78	188	0.3391	45.6	1706.1	1671.0	-35.1
76	231	155	79	188	0.3420	45.9	1708.1	1671.9	-36.3
76	232	156	80	188	0.3448	46.2	1710.1	1674.4	-35.7
76	233	157	81	188	0.3476	46.6	1711.9	1675.1	-36.8
76	234	158	82	188	0.3504	46.9	1713.6	1677.4	-36.2
76	235	159	83	188	0.3532	47.3	1715.2	1678.0	-37.3
76	236	160	84	188	0.3559	47.6	1716.7	1680.1	-36.6
76	237	161	85	188	0.3587	47.9	1718.0	1680.5	-37.6
76	238	162	86	188	0.3614	48.3	1719.2	1682.5	-36.8
76	239	163	87	188	0.3640	48.6	1720.3	1682.6	-37.7
76	240	164	88	188	0.3667	49.0	1721.3	1684.5	-36.9
76	241	165	89	188	0.3693	49.3	1722.2	1684.5	-37.7
76	242	166	90	188	0.3719	49.7	1722.9	1686.1	-36.8
76	243	167	91	188	0.3745	50.0	1723.6	1686.0	-37.6
76	244	168	92	188	0.3771	50.4	1724.1	1687.5	-36.6
76	245	169	93	188	0.3796	50.7	1724.5	1687.2	-37.3
76	246	170	94	188	0.3821	51.1	1724.7	1688.5	-36.2
76	247	171	95	188	0.3846	51.4	1724.9	1688.1	-36.8
76	248	172	96	188	0.3871	51.8	1724.9	1689.2	-35.7
76	249	173	97	188	0.3896	52.1	1724.8	1688.6	-36.2
76	250	174	98	188	0.3920	52.5	1724.6	1689.7	-34.9

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
81	158	77	-4	203	0.0253	32.1	1116.3	1117.4	1.1
81	159	78	-3	203	0.0189	32.1	1131.2	1133.7	2.6
81	160	79	-2	203	0.0125	32.0	1145.8	1148.0	2.2
81	161	80	-1	203	0.0062	32.0	1160.2	1163.8	3.6
81	162	81	0	203	0.0000	32.0	1174.4	1177.6	3.2
81	163	82	1	203	0.0061	32.0	1188.3	1192.8	4.5
81	164	83	2	203	0.0122	32.0	1202.1	1206.1	4.0
81	165	84	3	203	0.0182	32.0	1215.6	1220.8	5.2
81	166	85	4	203	0.0241	32.1	1228.9	1233.5	4.6
81	167	86	5	203	0.0299	32.1	1242.1	1247.8	5.7
81	168	87	6	203	0.0357	32.2	1255.0	1260.0	5.1
81	169	88	7	203	0.0414	32.3	1267.7	1273.8	6.1
81	170	89	8	203	0.0471	32.3	1280.3	1285.6	5.3
81	171	90	9	203	0.0526	32.4	1292.6	1298.9	6.3
81	172	91	10	203	0.0581	32.5	1304.8	1310.2	5.5
81	173	92	11	203	0.0636	32.6	1316.8	1323.0	6.3
81	174	93	12	203	0.0690	32.7	1328.6	1334.0	5.4
81	175	94	13	203	0.0743	32.9	1340.2	1346.3	6.1
81	176	95	14	203	0.0796	33.0	1351.7	1356.9	5.2
81	177	96	15	203	0.0848	33.1	1362.9	1368.8	5.9
81	178	97	16	203	0.0899	33.3	1374.1	1378.9	4.8
81	179	98	17	203	0.0950	33.4	1385.0	1390.4	5.4
81	180	99	18	203	0.1000	33.6	1395.8	1400.1	4.3
81	181	100	19	203	0.1050	33.7	1406.4	1411.2	4.8
81	182	101	20	203	0.1099	33.9	1416.9	1420.6	3.7
81	183	102	21	203	0.1148	34.0	1427.2	1431.3	4.1
81	184	103	22	203	0.1196	34.2	1437.4	1440.2	2.9
81	185	104	23	203	0.1243	34.4	1447.4	1450.6	3.2

81	186	105	24	203	0.1290	34.6	1457.2	1459.1	1.9
81	187	106	25	203	0.1337	34.8	1466.9	1469.1	2.2
81	188	107	26	203	0.1383	35.0	1476.4	1477.3	0.9
81	189	108	27	203	0.1429	35.2	1485.8	1487.0	1.1
81	190	109	28	203	0.1474	35.4	1495.1	1494.8	-0.2
81	191	110	29	203	0.1518	35.6	1504.2	1504.1	-0.1
81	192	111	30	203	0.1563	35.8	1513.1	1511.6	-1.5
81	193	112	31	203	0.1606	36.0	1521.9	1520.6	-1.4
81	194	113	32	203	0.1650	36.3	1530.6	1527.8	-2.8
81	195	114	33	203	0.1692	36.5	1539.1	1536.4	-2.8
81	196	115	34	203	0.1735	36.7	1547.5	1543.3	-4.3
81	197	116	35	203	0.1777	37.0	1555.8	1551.6	-4.2
81	198	117	36	203	0.1818	37.2	1563.9	1558.1	-5.8
81	199	118	37	203	0.1859	37.5	1571.9	1566.1	-5.8
81	200	119	38	203	0.1900	37.7	1579.7	1572.4	-7.3
81	201	120	39	203	0.1940	38.0	1587.4	1580.1	-7.4
81	202	121	40	203	0.1980	38.2	1595.0	1586.1	-8.9
81	203	122	41	203	0.2020	38.5	1602.4	1593.4	-9.0
81	204	123	42	203	0.2059	38.7	1609.7	1599.1	-10.6
81	205	124	43	203	0.2098	39.0	1616.9	1606.2	-10.7
81	206	125	44	203	0.2136	39.3	1623.9	1611.6	-12.3
81	207	126	45	203	0.2174	39.5	1630.8	1618.4	-12.4
81	208	127	46	203	0.2212	39.8	1637.6	1623.6	-14.0
81	209	128	47	203	0.2249	40.1	1644.3	1630.1	-14.2
81	210	129	48	203	0.2286	40.4	1650.8	1635.0	-15.8
81	211	130	49	203	0.2322	40.7	1657.2	1641.2	-15.9
81	212	131	50	203	0.2359	40.9	1663.4	1645.9	-17.6
81	213	132	51	203	0.2394	41.2	1669.6	1651.9	-17.7
81	214	133	52	203	0.2430	41.5	1675.6	1656.3	-19.3
81	215	134	53	203	0.2465	41.8	1681.5	1662.0	-19.5
81	216	135	54	203	0.2500	42.1	1687.2	1666.1	-21.1
81	217	136	55	203	0.2535	42.4	1692.9	1671.6	-21.3
81	218	137	56	203	0.2569	42.7	1698.4	1675.5	-22.9
81	219	138	57	203	0.2603	43.0	1703.8	1680.8	-23.0
81	220	139	58	203	0.2636	43.3	1709.0	1684.4	-24.6
81	221	140	59	203	0.2670	43.6	1714.2	1689.4	-24.8
81	222	141	60	203	0.2703	44.0	1719.2	1692.9	-26.3
81	223	142	61	203	0.2735	44.3	1724.1	1697.7	-26.5
81	224	143	62	203	0.2768	44.6	1728.9	1700.9	-28.0
81	225	144	63	203	0.2800	44.9	1733.5	1705.4	-28.1
81	226	145	64	203	0.2832	45.2	1738.1	1708.4	-29.7
81	227	146	65	203	0.2863	45.5	1742.5	1712.8	-29.7
81	228	147	66	203	0.2895	45.9	1746.8	1715.5	-31.3
81	229	148	67	203	0.2926	46.2	1751.0	1719.7	-31.3
81	230	149	68	203	0.2957	46.5	1755.0	1722.2	-32.8
81	231	150	69	203	0.2987	46.9	1759.0	1726.2	-32.8
81	232	151	70	203	0.3017	47.2	1762.8	1728.5	-34.3
81	233	152	71	203	0.3047	47.5	1766.5	1732.2	-34.3
81	234	153	72	203	0.3077	47.9	1770.1	1734.4	-35.7
81	235	154	73	203	0.3106	48.2	1773.6	1737.9	-35.7
81	236	155	74	203	0.3136	48.5	1776.9	1739.9	-37.0
81	237	156	75	203	0.3165	48.9	1780.1	1743.2	-36.9
81	238	157	76	203	0.3193	49.2	1783.3	1745.0	-38.3
81	239	158	77	203	0.3222	49.6	1786.3	1748.1	-38.2
81	240	159	78	203	0.3250	49.9	1789.1	1749.7	-39.4
81	241	160	79	203	0.3278	50.3	1791.9	1752.6	-39.3
81	242	161	80	203	0.3306	50.6	1794.6	1754.1	-40.5
81	243	162	81	203	0.3333	51.0	1797.1	1756.8	-40.3
81	244	163	82	203	0.3361	51.3	1799.5	1758.1	-41.5

81	245	164	83	203	0.3388	51.7	1801.8	1760.6	-41.2
81	246	165	84	203	0.3415	52.0	1804.0	1761.7	-42.3
81	247	166	85	203	0.3441	52.4	1806.1	1764.1	-42.0
81	248	167	86	203	0.3468	52.7	1808.1	1765.0	-43.1
81	249	168	87	203	0.3494	53.1	1809.9	1767.2	-42.7
81	250	169	88	203	0.3520	53.4	1811.7	1768.0	-43.7
81	251	170	89	203	0.3546	53.8	1813.3	1770.0	-43.3
81	252	171	90	203	0.3571	54.2	1814.8	1770.6	-44.2
81	253	172	91	203	0.3597	54.5	1816.2	1772.5	-43.8
81	254	173	92	203	0.3622	54.9	1817.5	1772.9	-44.6
81	255	174	93	203	0.3647	55.3	1818.7	1774.6	-44.1
81	256	175	94	203	0.3672	55.6	1819.8	1774.9	-44.9
81	257	176	95	203	0.3697	56.0	1820.7	1776.4	-44.3
81	258	177	96	203	0.3721	56.4	1821.5	1776.6	-45.0
81	259	178	97	203	0.3745	56.8	1822.3	1778.0	-44.3
81	260	179	98	203	0.3769	57.1	1822.9	1777.9	-45.0
81	261	180	99	203	0.3793	57.5	1823.4	1779.2	-44.2
81	262	181	100	203	0.3817	57.9	1823.8	1779.0	-44.8
81	263	182	101	203	0.3840	58.3	1824.1	1780.1	-44.0
81	264	183	102	203	0.3864	58.6	1824.2	1779.8	-44.5
81	265	184	103	203	0.3887	59.0	1824.3	1780.7	-43.6
81	266	185	104	203	0.3910	59.4	1824.2	1780.3	-44.0
81	267	186	105	203	0.3933	59.8	1824.1	1781.1	-43.0

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
86	168	82	-4	218	0.0238	36.1	1160.6	1166.0	5.4
86	169	83	-3	218	0.0178	36.1	1175.6	1180.4	4.8
86	170	84	-2	218	0.0118	36.0	1190.5	1196.8	6.3
86	171	85	-1	218	0.0059	36.0	1205.0	1210.7	5.7
86	172	86	0	218	0.0000	36.0	1219.4	1226.5	7.1
86	173	87	1	218	0.0058	36.0	1233.5	1240.0	6.4
86	174	88	2	218	0.0115	36.0	1247.5	1255.3	7.8
86	175	89	3	218	0.0171	36.1	1261.2	1268.2	7.0
86	176	90	4	218	0.0227	36.1	1274.7	1283.1	8.3
86	177	91	5	218	0.0283	36.1	1288.1	1295.6	7.5
86	178	92	6	218	0.0337	36.2	1301.2	1309.9	8.7
86	179	93	7	218	0.0391	36.3	1314.2	1322.0	7.8
86	180	94	8	218	0.0444	36.4	1327.0	1335.9	9.0
86	181	95	9	218	0.0497	36.4	1339.5	1347.5	8.0
86	182	96	10	218	0.0550	36.5	1352.0	1361.0	9.0
86	183	97	11	218	0.0601	36.6	1364.2	1372.2	8.0
86	184	98	12	218	0.0652	36.8	1376.2	1385.2	9.0
86	185	99	13	218	0.0703	36.9	1388.1	1396.0	7.9
86	186	100	14	218	0.0753	37.0	1399.9	1408.6	8.8
86	187	101	15	218	0.0802	37.1	1411.4	1419.0	7.6
86	188	102	16	218	0.0851	37.3	1422.8	1431.2	8.4
86	189	103	17	218	0.0900	37.4	1434.0	1441.2	7.2
86	190	104	18	218	0.0947	37.6	1445.1	1453.0	7.9
86	191	105	19	218	0.0995	37.7	1456.0	1462.6	6.6
86	192	106	20	218	0.1042	37.9	1466.8	1474.0	7.3
86	193	107	21	218	0.1088	38.1	1477.4	1483.3	5.9
86	194	108	22	218	0.1134	38.3	1487.8	1494.3	6.5
86	195	109	23	218	0.1180	38.5	1498.1	1503.2	5.1
86	196	110	24	218	0.1225	38.6	1508.3	1513.9	5.6
86	197	111	25	218	0.1269	38.8	1518.3	1522.5	4.2
86	198	112	26	218	0.1313	39.0	1528.2	1532.8	4.6
86	199	113	27	218	0.1357	39.2	1537.9	1541.0	3.1

86	200	114	28	218	0.1400	39.5	1547.4	1551.0	3.5
86	201	115	29	218	0.1443	39.7	1556.9	1558.9	2.0
86	202	116	30	218	0.1485	39.9	1566.2	1568.5	2.3
86	203	117	31	218	0.1527	40.1	1575.3	1576.1	0.8
86	204	118	32	218	0.1569	40.4	1584.3	1585.4	1.1
86	205	119	33	218	0.1610	40.6	1593.2	1592.6	-0.6
86	206	120	34	218	0.1651	40.8	1602.0	1601.6	-0.3
86	207	121	35	218	0.1691	41.1	1610.6	1608.6	-2.0
86	208	122	36	218	0.1731	41.3	1619.0	1617.3	-1.8
86	209	123	37	218	0.1770	41.6	1627.4	1623.9	-3.5
86	210	124	38	218	0.1810	41.8	1635.6	1632.3	-3.3
86	211	125	39	218	0.1848	42.1	1643.7	1638.7	-5.0
86	212	126	40	218	0.1887	42.3	1651.6	1646.8	-4.9
86	213	127	41	218	0.1925	42.6	1659.5	1652.8	-6.6
86	214	128	42	218	0.1963	42.9	1667.2	1660.6	-6.5
86	215	129	43	218	0.2000	43.2	1674.7	1666.4	-8.3
86	216	130	44	218	0.2037	43.4	1682.2	1674.0	-8.2
86	217	131	45	218	0.2074	43.7	1689.5	1679.5	-10.0
86	218	132	46	218	0.2110	44.0	1696.7	1686.7	-10.0
86	219	133	47	218	0.2146	44.3	1703.7	1692.0	-11.7
86	220	134	48	218	0.2182	44.6	1710.7	1699.0	-11.7
86	221	135	49	218	0.2217	44.9	1717.5	1704.0	-13.5
86	222	136	50	218	0.2252	45.2	1724.2	1710.7	-13.5
86	223	137	51	218	0.2287	45.5	1730.8	1715.5	-15.3
86	224	138	52	218	0.2321	45.8	1737.2	1721.9	-15.3
86	225	139	53	218	0.2356	46.1	1743.6	1726.5	-17.1
86	226	140	54	218	0.2389	46.4	1749.8	1732.7	-17.1
86	227	141	55	218	0.2423	46.7	1755.9	1737.0	-18.9
86	228	142	56	218	0.2456	47.0	1761.9	1742.9	-18.9
86	229	143	57	218	0.2489	47.3	1767.7	1747.0	-20.7
86	230	144	58	218	0.2522	47.6	1773.5	1752.7	-20.7
86	231	145	59	218	0.2554	47.9	1779.1	1756.6	-22.5
86	232	146	60	218	0.2586	48.3	1784.6	1762.1	-22.5
86	233	147	61	218	0.2618	48.6	1790.0	1765.7	-24.3
86	234	148	62	218	0.2650	48.9	1795.3	1770.9	-24.3
86	235	149	63	218	0.2681	49.2	1800.4	1774.3	-26.1
86	236	150	64	218	0.2712	49.6	1805.5	1779.4	-26.1
86	237	151	65	218	0.2743	49.9	1810.4	1782.5	-27.8
86	238	152	66	218	0.2773	50.3	1815.2	1787.4	-27.8
86	239	153	67	218	0.2803	50.6	1819.9	1790.4	-29.6
86	240	154	68	218	0.2833	50.9	1824.5	1795.0	-29.5
86	241	155	69	218	0.2863	51.3	1829.0	1797.7	-31.2
86	242	156	70	218	0.2893	51.6	1833.3	1802.2	-31.2
86	243	157	71	218	0.2922	52.0	1837.6	1804.7	-32.8
86	244	158	72	218	0.2951	52.3	1841.7	1808.9	-32.8
86	245	159	73	218	0.2980	52.7	1845.7	1811.3	-34.4
86	246	160	74	218	0.3008	53.0	1849.6	1815.3	-34.3
86	247	161	75	218	0.3036	53.4	1853.4	1817.5	-35.9
86	248	162	76	218	0.3065	53.7	1857.1	1821.3	-35.8
86	249	163	77	218	0.3092	54.1	1860.7	1823.3	-37.4
86	250	164	78	218	0.3120	54.4	1864.2	1827.0	-37.2
86	251	165	79	218	0.3147	54.8	1867.5	1828.8	-38.8
86	252	166	80	218	0.3175	55.2	1870.8	1832.2	-38.6
86	253	167	81	218	0.3202	55.5	1873.9	1833.9	-40.1
86	254	168	82	218	0.3228	55.9	1876.9	1837.1	-39.8
86	255	169	83	218	0.3255	56.3	1879.9	1838.6	-41.3
86	256	170	84	218	0.3281	56.6	1882.7	1841.7	-41.0
86	257	171	85	218	0.3307	57.0	1885.4	1843.0	-42.4
86	258	172	86	218	0.3333	57.4	1888.0	1845.9	-42.1

86	259	173	87	218	0.3359	57.7	1890.5	1847.0	-43.5
86	260	174	88	218	0.3385	58.1	1892.8	1849.7	-43.1
86	261	175	89	218	0.3410	58.5	1895.1	1850.7	-44.4
86	262	176	90	218	0.3435	58.9	1897.3	1853.3	-44.0
86	263	177	91	218	0.3460	59.3	1899.3	1854.1	-45.3
86	264	178	92	218	0.3485	59.6	1901.3	1856.5	-44.8
86	265	179	93	218	0.3509	60.0	1903.1	1857.1	-46.0
86	266	180	94	218	0.3534	60.4	1904.9	1859.4	-45.5
86	267	181	95	218	0.3558	60.8	1906.5	1859.9	-46.6
86	268	182	96	218	0.3582	61.2	1908.0	1861.9	-46.1
86	269	183	97	218	0.3606	61.6	1909.4	1862.3	-47.2
86	270	184	98	218	0.3630	62.0	1910.7	1864.2	-46.5
86	271	185	99	218	0.3653	62.3	1911.9	1864.4	-47.5
86	272	186	100	218	0.3677	62.7	1913.0	1866.2	-46.9
86	273	187	101	218	0.3700	63.1	1914.0	1866.2	-47.8
86	274	188	102	218	0.3723	63.5	1914.9	1867.8	-47.1
86	275	189	103	218	0.3746	63.9	1915.7	1867.8	-48.0
86	276	190	104	218	0.3768	64.3	1916.4	1869.2	-47.2
86	277	191	105	218	0.3791	64.7	1917.0	1869.0	-48.0
86	278	192	106	218	0.3813	65.1	1917.4	1870.3	-47.1
86	279	193	107	218	0.3835	65.5	1917.8	1870.0	-47.8
86	280	194	108	218	0.3857	65.9	1918.1	1871.2	-46.9
86	281	195	109	218	0.3879	66.3	1918.2	1870.7	-47.6
86	282	196	110	218	0.3901	66.7	1918.3	1871.7	-46.6
86	283	197	111	218	0.3922	67.1	1918.2	1871.1	-47.1

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
91	178	87	-4	231	0.0225	40.4	1210.6	1207.5	-3.1
91	179	88	-3	231	0.0168	40.3	1225.7	1224.0	-1.7
91	180	89	-2	231	0.0111	40.3	1240.5	1238.5	-2.0
91	181	90	-1	231	0.0055	40.3	1255.1	1254.4	-0.7
91	182	91	0	231	0.0000	40.3	1269.5	1268.5	-1.0
91	183	92	1	231	0.0055	40.3	1283.7	1283.9	0.3
91	184	93	2	231	0.0109	40.3	1297.6	1297.5	-0.1
91	185	94	3	231	0.0162	40.3	1311.4	1312.5	1.1
91	186	95	4	231	0.0215	40.3	1325.0	1325.6	0.7
91	187	96	5	231	0.0267	40.4	1338.4	1340.2	1.8
91	188	97	6	231	0.0319	40.5	1351.6	1352.8	1.3
91	189	98	7	231	0.0370	40.5	1364.6	1367.0	2.4
91	190	99	8	231	0.0421	40.6	1377.4	1379.2	1.8
91	191	100	9	231	0.0471	40.7	1390.1	1392.9	2.8
91	192	101	10	231	0.0521	40.8	1402.5	1404.7	2.2
91	193	102	11	231	0.0570	40.9	1414.8	1418.0	3.1
91	194	103	12	231	0.0619	41.0	1427.0	1429.4	2.4
91	195	104	13	231	0.0667	41.1	1438.9	1442.2	3.3
91	196	105	14	231	0.0714	41.3	1450.7	1453.3	2.6
91	197	106	15	231	0.0761	41.4	1462.4	1465.7	3.4
91	198	107	16	231	0.0808	41.5	1473.9	1476.4	2.5
91	199	108	17	231	0.0854	41.7	1485.2	1488.5	3.3
91	200	109	18	231	0.0900	41.8	1496.3	1498.7	2.4
91	201	110	19	231	0.0945	42.0	1507.3	1510.4	3.1
91	202	111	20	231	0.0990	42.2	1518.2	1520.4	2.2
91	203	112	21	231	0.1035	42.4	1528.9	1531.7	2.8
91	204	113	22	231	0.1078	42.5	1539.5	1541.3	1.8
91	205	114	23	231	0.1122	42.7	1549.9	1552.2	2.3
91	206	115	24	231	0.1165	42.9	1560.1	1561.5	1.3
91	207	116	25	231	0.1208	43.1	1570.3	1572.1	1.8

91	208	117	26	231	0.1250	43.3	1580.3	1581.0	0.7
91	209	118	27	231	0.1292	43.5	1590.1	1591.3	1.2
91	210	119	28	231	0.1333	43.8	1599.8	1599.8	0.0
91	211	120	29	231	0.1374	44.0	1609.4	1609.8	0.4
91	212	121	30	231	0.1415	44.2	1618.8	1618.0	-0.7
91	213	122	31	231	0.1455	44.4	1628.1	1627.7	-0.4
91	214	123	32	231	0.1495	44.7	1637.2	1635.6	-1.6
91	215	124	33	231	0.1535	44.9	1646.2	1644.9	-1.3
91	216	125	34	231	0.1574	45.2	1655.1	1652.6	-2.6
91	217	126	35	231	0.1613	45.4	1663.9	1661.6	-2.3
91	218	127	36	231	0.1651	45.7	1672.5	1668.9	-3.6
91	219	128	37	231	0.1690	45.9	1681.0	1677.6	-3.4
91	220	129	38	231	0.1727	46.2	1689.4	1684.7	-4.7
91	221	130	39	231	0.1765	46.4	1697.6	1693.1	-4.5
91	222	131	40	231	0.1802	46.7	1705.7	1699.9	-5.9
91	223	132	41	231	0.1839	47.0	1713.7	1708.0	-5.7
91	224	133	42	231	0.1875	47.3	1721.6	1714.5	-7.1
91	225	134	43	231	0.1911	47.5	1729.3	1722.4	-7.0
91	226	135	44	231	0.1947	47.8	1737.0	1728.6	-8.3
91	227	136	45	231	0.1982	48.1	1744.5	1736.2	-8.2
91	228	137	46	231	0.2018	48.4	1751.8	1742.2	-9.6
91	229	138	47	231	0.2052	48.7	1759.1	1749.5	-9.6
91	230	139	48	231	0.2087	49.0	1766.2	1755.2	-11.0
91	231	140	49	231	0.2121	49.3	1773.2	1762.3	-10.9
91	232	141	50	231	0.2155	49.6	1780.1	1767.8	-12.4
91	233	142	51	231	0.2189	49.9	1786.9	1774.6	-12.3
91	234	143	52	231	0.2222	50.2	1793.6	1779.8	-13.8
91	235	144	53	231	0.2255	50.5	1800.1	1786.4	-13.7
91	236	145	54	231	0.2288	50.9	1806.5	1791.4	-15.2
91	237	146	55	231	0.2321	51.2	1812.9	1797.7	-15.2
91	238	147	56	231	0.2353	51.5	1819.0	1802.5	-16.6
91	239	148	57	231	0.2385	51.8	1825.1	1808.5	-16.6
91	240	149	58	231	0.2417	52.1	1831.1	1813.1	-18.0
91	241	150	59	231	0.2448	52.5	1836.9	1818.9	-18.0
91	242	151	60	231	0.2479	52.8	1842.7	1823.2	-19.5
91	243	152	61	231	0.2510	53.1	1848.3	1828.9	-19.5
91	244	153	62	231	0.2541	53.5	1853.8	1833.0	-20.9
91	245	154	63	231	0.2571	53.8	1859.2	1838.4	-20.9
91	246	155	64	231	0.2602	54.2	1864.5	1842.2	-22.3
91	247	156	65	231	0.2632	54.5	1869.7	1847.4	-22.3
91	248	157	66	231	0.2661	54.9	1874.8	1851.1	-23.7
91	249	158	67	231	0.2691	55.2	1879.7	1856.1	-23.7
91	250	159	68	231	0.2720	55.6	1884.6	1859.5	-25.0
91	251	160	69	231	0.2749	55.9	1889.3	1864.3	-25.0
91	252	161	70	231	0.2778	56.3	1893.9	1867.6	-26.4
91	253	162	71	231	0.2806	56.6	1898.5	1872.1	-26.3
91	254	163	72	231	0.2835	57.0	1902.9	1875.2	-27.7
91	255	164	73	231	0.2863	57.3	1907.2	1879.6	-27.6
91	256	165	74	231	0.2891	57.7	1911.4	1882.5	-28.9
91	257	166	75	231	0.2918	58.1	1915.5	1886.6	-28.8
91	258	167	76	231	0.2946	58.4	1919.4	1889.3	-30.1
91	259	168	77	231	0.2973	58.8	1923.3	1893.3	-30.0
91	260	169	78	231	0.3000	59.2	1927.1	1895.8	-31.3
91	261	170	79	231	0.3027	59.6	1930.8	1899.6	-31.1
91	262	171	80	231	0.3053	59.9	1934.3	1902.0	-32.4
91	263	172	81	231	0.3080	60.3	1937.8	1905.6	-32.2
91	264	173	82	231	0.3106	60.7	1941.1	1907.7	-33.4
91	265	174	83	231	0.3132	61.1	1944.3	1911.2	-33.2
91	266	175	84	231	0.3158	61.5	1947.5	1913.1	-34.4

91	267	176	85	231	0.3184	61.8	1950.5	1916.4	-34.1
91	268	177	86	231	0.3209	62.2	1953.4	1918.2	-35.2
91	269	178	87	231	0.3234	62.6	1956.3	1921.3	-35.0
91	270	179	88	231	0.3259	63.0	1959.0	1922.9	-36.1
91	271	180	89	231	0.3284	63.4	1961.6	1925.9	-35.7
91	272	181	90	231	0.3309	63.8	1964.1	1927.3	-36.8
91	273	182	91	231	0.3333	64.2	1966.5	1930.1	-36.4
91	274	183	92	231	0.3358	64.6	1968.8	1931.4	-37.4
91	275	184	93	231	0.3382	65.0	1971.0	1934.0	-37.0
91	276	185	94	231	0.3406	65.4	1973.1	1935.2	-38.0
91	277	186	95	231	0.3430	65.8	1975.1	1937.6	-37.5
91	278	187	96	231	0.3453	66.2	1977.0	1938.6	-38.4
91	279	188	97	231	0.3477	66.6	1978.8	1940.9	-37.9
91	280	189	98	231	0.3500	67.0	1980.5	1941.8	-38.8
91	281	190	99	231	0.3523	67.4	1982.1	1943.9	-38.2
91	282	191	100	231	0.3546	67.8	1983.6	1944.6	-39.0
91	283	192	101	231	0.3569	68.2	1985.0	1946.6	-38.4
91	284	193	102	231	0.3592	68.6	1986.3	1947.1	-39.2
91	285	194	103	231	0.3614	69.0	1987.5	1948.9	-38.5
91	286	195	104	231	0.3636	69.4	1988.6	1949.4	-39.2
91	287	196	105	231	0.3659	69.8	1989.5	1951.1	-38.5
91	288	197	106	231	0.3681	70.3	1990.4	1951.3	-39.1
91	289	198	107	231	0.3702	70.7	1991.2	1952.9	-38.3
91	290	199	108	231	0.3724	71.1	1991.9	1953.0	-38.9
91	291	200	109	231	0.3746	71.5	1992.5	1954.4	-38.1
91	292	201	110	231	0.3767	71.9	1993.0	1954.4	-38.5
91	293	202	111	231	0.3788	72.4	1993.4	1955.7	-37.7
91	294	203	112	231	0.3810	72.8	1993.7	1955.6	-38.1
91	295	204	113	231	0.3831	73.2	1993.8	1956.7	-37.1
91	296	205	114	231	0.3851	73.6	1993.9	1956.5	-37.5
91	297	206	115	231	0.3872	74.1	1993.9	1957.5	-36.5
91	298	207	116	231	0.3893	74.5	1993.8	1957.1	-36.7
91	299	208	117	231	0.3913	74.9	1993.6	1957.9	-35.7
91	300	209	118	231	0.3933	75.3	1993.3	1957.4	-35.9

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
96	188	92	-4	246	0.0213	44.8	1250.0	1249.8	-0.2
96	189	93	-3	246	0.0159	44.8	1265.1	1264.5	-0.7
96	190	94	-2	246	0.0105	44.8	1280.1	1280.9	0.8
96	191	95	-1	246	0.0052	44.7	1294.8	1295.1	0.3
96	192	96	0	246	0.0000	44.7	1309.4	1311.1	1.8
96	193	97	1	246	0.0052	44.7	1323.7	1324.9	1.2
96	194	98	2	246	0.0103	44.8	1337.8	1340.4	2.6
96	195	99	3	246	0.0154	44.8	1351.8	1353.7	1.9
96	196	100	4	246	0.0204	44.8	1365.5	1368.8	3.3
96	197	101	5	246	0.0254	44.9	1379.1	1381.7	2.6
96	198	102	6	246	0.0303	44.9	1392.5	1396.4	3.9
96	199	103	7	246	0.0352	45.0	1405.7	1408.8	3.2
96	200	104	8	246	0.0400	45.1	1418.7	1423.1	4.4
96	201	105	9	246	0.0448	45.2	1431.5	1435.1	3.6
96	202	106	10	246	0.0495	45.3	1444.2	1449.0	4.8
96	203	107	11	246	0.0542	45.4	1456.7	1460.6	3.9
96	204	108	12	246	0.0588	45.5	1469.0	1474.1	5.1
96	205	109	13	246	0.0634	45.6	1481.2	1485.4	4.2
96	206	110	14	246	0.0680	45.8	1493.2	1498.4	5.2
96	207	111	15	246	0.0725	45.9	1505.1	1509.3	4.3
96	208	112	16	246	0.0769	46.0	1516.8	1522.0	5.2

96	209	113	17	246	0.0813	46.2	1528.3	1532.6	4.3
96	210	114	18	246	0.0857	46.3	1539.7	1544.9	5.2
96	211	115	19	246	0.0901	46.5	1550.9	1555.1	4.1
96	212	116	20	246	0.0943	46.7	1562.0	1567.0	5.0
96	213	117	21	246	0.0986	46.9	1573.0	1576.9	3.9
96	214	118	22	246	0.1028	47.1	1583.8	1588.5	4.7
96	215	119	23	246	0.1070	47.2	1594.4	1598.0	3.6
96	216	120	24	246	0.1111	47.4	1604.9	1609.2	4.3
96	217	121	25	246	0.1152	47.6	1615.3	1618.4	3.1
96	218	122	26	246	0.1193	47.9	1625.5	1629.3	3.8
96	219	123	27	246	0.1233	48.1	1635.6	1638.2	2.6
96	220	124	28	246	0.1273	48.3	1645.6	1648.8	3.2
96	221	125	29	246	0.1312	48.5	1655.4	1657.4	2.0
96	222	126	30	246	0.1351	48.7	1665.1	1667.6	2.5
96	223	127	31	246	0.1390	49.0	1674.7	1675.9	1.2
96	224	128	32	246	0.1429	49.2	1684.1	1685.8	1.8
96	225	129	33	246	0.1467	49.5	1693.4	1693.8	0.4
96	226	130	34	246	0.1504	49.7	1702.5	1703.5	0.9
96	227	131	35	246	0.1542	50.0	1711.6	1711.1	-0.5
96	228	132	36	246	0.1579	50.2	1720.5	1720.5	0.0
96	229	133	37	246	0.1616	50.5	1729.3	1727.9	-1.4
96	230	134	38	246	0.1652	50.8	1737.9	1737.0	-1.0
96	231	135	39	246	0.1688	51.0	1746.5	1744.1	-2.4
96	232	136	40	246	0.1724	51.3	1754.9	1752.9	-2.0
96	233	137	41	246	0.1760	51.6	1763.2	1759.7	-3.5
96	234	138	42	246	0.1795	51.9	1771.4	1768.2	-3.2
96	235	139	43	246	0.1830	52.2	1779.4	1774.8	-4.6
96	236	140	44	246	0.1864	52.5	1787.3	1783.0	-4.3
96	237	141	45	246	0.1899	52.7	1795.2	1789.3	-5.8
96	238	142	46	246	0.1933	53.0	1802.9	1797.3	-5.5
96	239	143	47	246	0.1967	53.3	1810.4	1803.4	-7.1
96	240	144	48	246	0.2000	53.7	1817.9	1811.1	-6.8
96	241	145	49	246	0.2033	54.0	1825.2	1816.9	-8.3
96	242	146	50	246	0.2066	54.3	1832.5	1824.4	-8.1
96	243	147	51	246	0.2099	54.6	1839.6	1830.0	-9.6
96	244	148	52	246	0.2131	54.9	1846.6	1837.2	-9.4
96	245	149	53	246	0.2163	55.2	1853.5	1842.5	-11.0
96	246	150	54	246	0.2195	55.6	1860.3	1849.5	-10.7
96	247	151	55	246	0.2227	55.9	1866.9	1854.6	-12.3
96	248	152	56	246	0.2258	56.2	1873.5	1861.4	-12.1
96	249	153	57	246	0.2289	56.5	1879.9	1866.2	-13.7
96	250	154	58	246	0.2320	56.9	1886.2	1872.8	-13.5
96	251	155	59	246	0.2351	57.2	1892.5	1877.4	-15.0
96	252	156	60	246	0.2381	57.6	1898.6	1883.7	-14.9
96	253	157	61	246	0.2411	57.9	1904.6	1888.2	-16.4
96	254	158	62	246	0.2441	58.3	1910.5	1894.2	-16.3
96	255	159	63	246	0.2471	58.6	1916.3	1898.5	-17.8
96	256	160	64	246	0.2500	59.0	1921.9	1904.3	-17.6
96	257	161	65	246	0.2529	59.3	1927.5	1908.3	-19.2
96	258	162	66	246	0.2558	59.7	1933.0	1914.0	-19.0
96	259	163	67	246	0.2587	60.0	1938.3	1917.8	-20.5
96	260	164	68	246	0.2615	60.4	1943.6	1923.2	-20.4
96	261	165	69	246	0.2644	60.8	1948.7	1926.8	-21.9
96	262	166	70	246	0.2672	61.1	1953.8	1932.1	-21.7
96	263	167	71	246	0.2700	61.5	1958.7	1935.5	-23.2
96	264	168	72	246	0.2727	61.9	1963.5	1940.5	-23.0
96	265	169	73	246	0.2755	62.2	1968.3	1943.7	-24.5
96	266	170	74	246	0.2782	62.6	1972.9	1948.6	-24.3
96	267	171	75	246	0.2809	63.0	1977.4	1951.6	-25.8

96	268	172	76	246	0.2836	63.4	1981.8	1956.3	-25.6
96	269	173	77	246	0.2863	63.8	1986.1	1959.1	-27.0
96	270	174	78	246	0.2889	64.1	1990.3	1963.6	-26.8
96	271	175	79	246	0.2915	64.5	1994.4	1966.2	-28.2
96	272	176	80	246	0.2941	64.9	1998.4	1970.5	-27.9
96	273	177	81	246	0.2967	65.3	2002.4	1973.0	-29.3
96	274	178	82	246	0.2993	65.7	2006.2	1977.1	-29.1
96	275	179	83	246	0.3018	66.1	2009.9	1979.4	-30.4
96	276	180	84	246	0.3044	66.5	2013.5	1983.3	-30.1
96	277	181	85	246	0.3069	66.9	2017.0	1985.5	-31.5
96	278	182	86	246	0.3094	67.3	2020.4	1989.2	-31.1
96	279	183	87	246	0.3118	67.7	2023.7	1991.2	-32.5
96	280	184	88	246	0.3143	68.1	2026.9	1994.8	-32.1
96	281	185	89	246	0.3167	68.5	2030.0	1996.6	-33.4
96	282	186	90	246	0.3192	68.9	2033.0	2000.0	-33.0
96	283	187	91	246	0.3216	69.3	2035.9	2001.7	-34.2
96	284	188	92	246	0.3239	69.7	2038.7	2004.9	-33.8
96	285	189	93	246	0.3263	70.1	2041.4	2006.4	-35.0
96	286	190	94	246	0.3287	70.5	2044.0	2009.5	-34.5
96	287	191	95	246	0.3310	70.9	2046.5	2010.9	-35.7
96	288	192	96	246	0.3333	71.4	2048.9	2013.8	-35.2
96	289	193	97	246	0.3356	71.8	2051.3	2015.0	-36.3
96	290	194	98	246	0.3379	72.2	2053.5	2017.7	-35.8
96	291	195	99	246	0.3402	72.6	2055.6	2018.8	-36.8
96	292	196	100	246	0.3425	73.0	2057.6	2021.4	-36.2
96	293	197	101	246	0.3447	73.5	2059.5	2022.3	-37.3
96	294	198	102	246	0.3469	73.9	2061.4	2024.8	-36.6
96	295	199	103	246	0.3492	74.3	2063.1	2025.5	-37.6
96	296	200	104	246	0.3514	74.7	2064.8	2027.8	-36.9
96	297	201	105	246	0.3535	75.2	2066.3	2028.5	-37.8
96	298	202	106	246	0.3557	75.6	2067.7	2030.6	-37.1
96	299	203	107	246	0.3579	76.0	2069.1	2031.1	-38.0
96	300	204	108	246	0.3600	76.5	2070.4	2033.1	-37.2
96	301	205	109	246	0.3621	76.9	2071.5	2033.5	-38.0
96	302	206	110	246	0.3642	77.3	2072.6	2035.4	-37.2
96	303	207	111	246	0.3663	77.8	2073.5	2035.6	-38.0
96	304	208	112	246	0.3684	78.2	2074.4	2037.3	-37.1
96	305	209	113	246	0.3705	78.7	2075.2	2037.4	-37.8
96	306	210	114	246	0.3726	79.1	2075.9	2039.0	-36.9
96	307	211	115	246	0.3746	79.5	2076.5	2039.0	-37.5
96	308	212	116	246	0.3766	80.0	2077.0	2040.4	-36.5
96	309	213	117	246	0.3786	80.4	2077.4	2040.3	-37.1
96	310	214	118	246	0.3807	80.9	2077.7	2041.6	-36.1
96	311	215	119	246	0.3826	81.3	2077.9	2041.3	-36.6
96	312	216	120	246	0.3846	81.8	2078.0	2042.5	-35.5
96	313	217	121	246	0.3866	82.2	2078.1	2042.1	-35.9
96	314	218	122	246	0.3885	82.7	2078.0	2043.2	-34.8
96	315	219	123	246	0.3905	83.1	2077.8	2042.7	-35.1
96	316	220	124	246	0.3924	83.6	2077.6	2043.7	-33.9

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
101	198	97	-4	261	0.0202	49.6	1286.7	1285.3	-1.4
101	199	98	-3	261	0.0151	49.5	1302.0	1301.9	-0.2
101	200	99	-2	261	0.0100	49.5	1317.1	1316.6	-0.5
101	201	100	-1	261	0.0050	49.5	1332.0	1332.7	0.7
101	202	101	0	261	0.0000	49.5	1346.6	1347.0	0.4
101	203	102	1	261	0.0049	49.5	1361.1	1362.7	1.6

101	204	103	2	261	0.0098	49.5	1375.4	1376.5	1.1
101	205	104	3	261	0.0146	49.5	1389.5	1391.8	2.3
101	206	105	4	261	0.0194	49.6	1403.4	1405.2	1.8
101	207	106	5	261	0.0242	49.6	1417.1	1420.1	3.0
101	208	107	6	261	0.0289	49.7	1430.6	1433.1	2.5
101	209	108	7	261	0.0335	49.7	1444.0	1447.5	3.5
101	210	109	8	261	0.0381	49.8	1457.2	1460.1	3.0
101	211	110	9	261	0.0427	49.9	1470.2	1474.2	4.0
101	212	111	10	261	0.0472	50.0	1483.0	1486.4	3.4
101	213	112	11	261	0.0516	50.1	1495.7	1500.1	4.4
101	214	113	12	261	0.0561	50.2	1508.2	1511.9	3.7
101	215	114	13	261	0.0605	50.4	1520.6	1525.2	4.6
101	216	115	14	261	0.0648	50.5	1532.8	1536.7	3.9
101	217	116	15	261	0.0691	50.6	1544.8	1549.6	4.8
101	218	117	16	261	0.0734	50.8	1556.7	1560.8	4.1
101	219	118	17	261	0.0776	50.9	1568.4	1573.3	4.9
101	220	119	18	261	0.0818	51.1	1580.0	1584.1	4.1
101	221	120	19	261	0.0860	51.3	1591.5	1596.3	4.8
101	222	121	20	261	0.0901	51.4	1602.8	1606.7	4.0
101	223	122	21	261	0.0942	51.6	1613.9	1618.6	4.7
101	224	123	22	261	0.0982	51.8	1624.9	1628.7	3.8
101	225	124	23	261	0.1022	52.0	1635.8	1640.2	4.4
101	226	125	24	261	0.1062	52.2	1646.5	1650.0	3.5
101	227	126	25	261	0.1101	52.4	1657.1	1661.2	4.1
101	228	127	26	261	0.1140	52.6	1667.5	1670.7	3.2
101	229	128	27	261	0.1179	52.8	1677.9	1681.6	3.7
101	230	129	28	261	0.1217	53.1	1688.1	1690.7	2.7
101	231	130	29	261	0.1255	53.3	1698.1	1701.3	3.2
101	232	131	30	261	0.1293	53.5	1708.0	1710.2	2.2
101	233	132	31	261	0.1331	53.8	1717.8	1720.4	2.6
101	234	133	32	261	0.1368	54.0	1727.5	1729.0	1.5
101	235	134	33	261	0.1404	54.3	1737.0	1738.9	1.9
101	236	135	34	261	0.1441	54.5	1746.4	1747.3	0.8
101	237	136	35	261	0.1477	54.8	1755.7	1756.9	1.2
101	238	137	36	261	0.1513	55.0	1764.9	1764.9	0.0
101	239	138	37	261	0.1548	55.3	1773.9	1774.3	0.4
101	240	139	38	261	0.1583	55.6	1782.8	1782.0	-0.8
101	241	140	39	261	0.1618	55.9	1791.6	1791.1	-0.5
101	242	141	40	261	0.1653	56.1	1800.3	1798.6	-1.7
101	243	142	41	261	0.1687	56.4	1808.9	1807.4	-1.5
101	244	143	42	261	0.1721	56.7	1817.3	1814.6	-2.7
101	245	144	43	261	0.1755	57.0	1825.6	1823.2	-2.5
101	246	145	44	261	0.1789	57.3	1833.8	1830.1	-3.7
101	247	146	45	261	0.1822	57.6	1841.9	1838.4	-3.5
101	248	147	46	261	0.1855	57.9	1849.9	1845.1	-4.8
101	249	148	47	261	0.1888	58.2	1857.8	1853.2	-4.6
101	250	149	48	261	0.1920	58.5	1865.5	1859.6	-5.9
101	251	150	49	261	0.1952	58.9	1873.1	1867.4	-5.7
101	252	151	50	261	0.1984	59.2	1880.7	1873.6	-7.0
101	253	152	51	261	0.2016	59.5	1888.1	1881.2	-6.9
101	254	153	52	261	0.2047	59.8	1895.4	1887.2	-8.2
101	255	154	53	261	0.2078	60.2	1902.6	1894.4	-8.1
101	256	155	54	261	0.2109	60.5	1909.6	1900.2	-9.4
101	257	156	55	261	0.2140	60.8	1916.6	1907.3	-9.4
101	258	157	56	261	0.2171	61.2	1923.5	1912.8	-10.7
101	259	158	57	261	0.2201	61.5	1930.2	1919.6	-10.6
101	260	159	58	261	0.2231	61.8	1936.9	1924.9	-11.9
101	261	160	59	261	0.2261	62.2	1943.4	1931.5	-11.9
101	262	161	60	261	0.2290	62.5	1949.9	1936.6	-13.2

101	263	162	61	261	0.2319	62.9	1956.2	1943.0	-13.2
101	264	163	62	261	0.2349	63.3	1962.4	1947.9	-14.5
101	265	164	63	261	0.2377	63.6	1968.5	1954.1	-14.5
101	266	165	64	261	0.2406	64.0	1974.5	1958.7	-15.8
101	267	166	65	261	0.2435	64.3	1980.4	1964.7	-15.7
101	268	167	66	261	0.2463	64.7	1986.2	1969.2	-17.1
101	269	168	67	261	0.2491	65.1	1991.9	1974.9	-17.0
101	270	169	68	261	0.2519	65.5	1997.5	1979.2	-18.4
101	271	170	69	261	0.2546	65.8	2003.0	1984.7	-18.3
101	272	171	70	261	0.2574	66.2	2008.4	1988.8	-19.6
101	273	172	71	261	0.2601	66.6	2013.7	1994.1	-19.6
101	274	173	72	261	0.2628	67.0	2018.9	1998.0	-20.9
101	275	174	73	261	0.2655	67.4	2024.0	2003.2	-20.8
101	276	175	74	261	0.2681	67.7	2029.0	2006.9	-22.1
101	277	176	75	261	0.2708	68.1	2033.9	2011.8	-22.1
101	278	177	76	261	0.2734	68.5	2038.7	2015.3	-23.4
101	279	178	77	261	0.2760	68.9	2043.4	2020.1	-23.3
101	280	179	78	261	0.2786	69.3	2048.0	2023.4	-24.6
101	281	180	79	261	0.2811	69.7	2052.5	2028.0	-24.5
101	282	181	80	261	0.2837	70.1	2056.9	2031.2	-25.7
101	283	182	81	261	0.2862	70.5	2061.2	2035.6	-25.6
101	284	183	82	261	0.2887	70.9	2065.4	2038.5	-26.8
101	285	184	83	261	0.2912	71.3	2069.5	2042.8	-26.7
101	286	185	84	261	0.2937	71.7	2073.5	2045.6	-27.9
101	287	186	85	261	0.2962	72.1	2077.4	2049.6	-27.8
101	288	187	86	261	0.2986	72.5	2081.2	2052.2	-29.0
101	289	188	87	261	0.3010	73.0	2084.9	2056.1	-28.8
101	290	189	88	261	0.3035	73.4	2088.5	2058.6	-29.9
101	291	190	89	261	0.3058	73.8	2092.1	2062.3	-29.7
101	292	191	90	261	0.3082	74.2	2095.5	2064.6	-30.9
101	293	192	91	261	0.3106	74.6	2098.8	2068.2	-30.7
101	294	193	92	261	0.3129	75.1	2102.1	2070.3	-31.8
101	295	194	93	261	0.3153	75.5	2105.2	2073.7	-31.5
101	296	195	94	261	0.3176	75.9	2108.3	2075.7	-32.6
101	297	196	95	261	0.3199	76.3	2111.2	2078.9	-32.3
101	298	197	96	261	0.3222	76.8	2114.1	2080.8	-33.3
101	299	198	97	261	0.3244	77.2	2116.8	2083.8	-33.0
101	300	199	98	261	0.3267	77.6	2119.5	2085.5	-34.0
101	301	200	99	261	0.3289	78.1	2122.1	2088.4	-33.7
101	302	201	100	261	0.3311	78.5	2124.6	2090.0	-34.6
101	303	202	101	261	0.3333	78.9	2127.0	2092.7	-34.2
101	304	203	102	261	0.3355	79.4	2129.3	2094.1	-35.1
101	305	204	103	261	0.3377	79.8	2131.5	2096.8	-34.7
101	306	205	104	261	0.3399	80.3	2133.6	2098.0	-35.6
101	307	206	105	261	0.3420	80.7	2135.6	2100.5	-35.1
101	308	207	106	261	0.3442	81.2	2137.5	2101.6	-36.0
101	309	208	107	261	0.3463	81.6	2139.4	2103.9	-35.5
101	310	209	108	261	0.3484	82.0	2141.1	2104.9	-36.3
101	311	210	109	261	0.3505	82.5	2142.8	2107.1	-35.7
101	312	211	110	261	0.3526	82.9	2144.4	2107.9	-36.4
101	313	212	111	261	0.3546	83.4	2145.8	2110.0	-35.9
101	314	213	112	261	0.3567	83.9	2147.2	2110.7	-36.5
101	315	214	113	261	0.3587	84.3	2148.5	2112.6	-35.9
101	316	215	114	261	0.3608	84.8	2149.7	2113.1	-36.6
101	317	216	115	261	0.3628	85.2	2150.8	2114.9	-35.9
101	318	217	116	261	0.3648	85.7	2151.8	2115.4	-36.5
101	319	218	117	261	0.3668	86.1	2152.8	2117.0	-35.7
101	320	219	118	261	0.3688	86.6	2153.6	2117.3	-36.3
101	321	220	119	261	0.3707	87.1	2154.3	2118.9	-35.5

101	322	221	120	261	0.3727	87.5	2155.0	2119.1	-36.0
101	323	222	121	261	0.3746	88.0	2155.6	2120.5	-35.1
101	324	223	122	261	0.3765	88.5	2156.1	2120.5	-35.5
101	325	224	123	261	0.3785	88.9	2156.4	2121.8	-34.7
101	326	225	124	261	0.3804	89.4	2156.7	2121.7	-35.0
101	327	226	125	261	0.3823	89.9	2157.0	2122.9	-34.1
101	328	227	126	261	0.3842	90.3	2157.1	2122.7	-34.4
101	329	228	127	261	0.3860	90.8	2157.1	2123.7	-33.4
101	330	229	128	261	0.3879	91.3	2157.1	2123.4	-33.6
101	331	230	129	261	0.3897	91.8	2156.9	2124.4	-32.6
101	332	231	130	261	0.3916	92.2	2156.7	2124.0	-32.7
101	333	232	131	261	0.3934	92.7	2156.4	2124.7	-31.6
Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
106	208	102	-4	278	0.0192	54.5	1312.2	1321.5	9.3
106	209	103	-3	278	0.0144	54.5	1327.7	1336.3	8.6
106	210	104	-2	278	0.0095	54.5	1343.0	1352.9	10.0
106	211	105	-1	278	0.0047	54.4	1358.1	1367.3	9.3
106	212	106	0	278	0.0000	54.4	1373.0	1383.5	10.6
106	213	107	1	278	0.0047	54.4	1387.7	1397.5	9.8
106	214	108	2	278	0.0094	54.5	1402.2	1413.3	11.1
106	215	109	3	278	0.0140	54.5	1416.5	1426.9	10.4
106	216	110	4	278	0.0185	54.5	1430.6	1442.2	11.6
106	217	111	5	278	0.0230	54.6	1444.6	1455.4	10.8
106	218	112	6	278	0.0275	54.6	1458.4	1470.4	12.0
106	219	113	7	278	0.0320	54.7	1472.0	1483.2	11.2
106	220	114	8	278	0.0364	54.8	1485.5	1497.7	12.3
106	221	115	9	278	0.0407	54.9	1498.8	1510.2	11.4
106	222	116	10	278	0.0451	55.0	1511.9	1524.4	12.5
106	223	117	11	278	0.0493	55.1	1524.8	1536.5	11.6
106	224	118	12	278	0.0536	55.2	1537.6	1550.3	12.7
106	225	119	13	278	0.0578	55.3	1550.3	1562.0	11.7
106	226	120	14	278	0.0620	55.5	1562.7	1575.4	12.7
106	227	121	15	278	0.0661	55.6	1575.1	1586.8	11.8
106	228	122	16	278	0.0702	55.8	1587.2	1599.9	12.7
106	229	123	17	278	0.0742	55.9	1599.3	1610.9	11.7
106	230	124	18	278	0.0783	56.1	1611.2	1623.7	12.5
106	231	125	19	278	0.0823	56.2	1622.9	1634.4	11.5
106	232	126	20	278	0.0862	56.4	1634.5	1646.8	12.3
106	233	127	21	278	0.0901	56.6	1645.9	1657.2	11.2
106	234	128	22	278	0.0940	56.8	1657.3	1669.3	12.0
106	235	129	23	278	0.0979	57.0	1668.4	1679.3	10.9
106	236	130	24	278	0.1017	57.2	1679.5	1691.1	11.6
106	237	131	25	278	0.1055	57.4	1690.4	1700.8	10.5
106	238	132	26	278	0.1092	57.6	1701.2	1712.3	11.1
106	239	133	27	278	0.1130	57.8	1711.8	1721.7	9.9
106	240	134	28	278	0.1167	58.1	1722.3	1732.9	10.6
106	241	135	29	278	0.1203	58.3	1732.7	1742.0	9.3
106	242	136	30	278	0.1240	58.5	1743.0	1752.9	9.9
106	243	137	31	278	0.1276	58.8	1753.1	1761.7	8.6
106	244	138	32	278	0.1312	59.0	1763.1	1772.3	9.2
106	245	139	33	278	0.1347	59.3	1773.0	1780.8	7.9
106	246	140	34	278	0.1382	59.5	1782.7	1791.1	8.4
106	247	141	35	278	0.1417	59.8	1792.4	1799.4	7.0
106	248	142	36	278	0.1452	60.1	1801.9	1809.4	7.5
106	249	143	37	278	0.1486	60.4	1811.3	1817.4	6.1
106	250	144	38	278	0.1520	60.6	1820.5	1827.1	6.6

106	251	145	39	278	0.1554	60.9	1829.7	1834.9	5.2
106	252	146	40	278	0.1587	61.2	1838.7	1844.3	5.6
106	253	147	41	278	0.1621	61.5	1847.7	1851.8	4.1
106	254	148	42	278	0.1654	61.8	1856.5	1860.9	4.5
106	255	149	43	278	0.1686	62.1	1865.2	1868.2	3.0
106	256	150	44	278	0.1719	62.4	1873.7	1877.1	3.4
106	257	151	45	278	0.1751	62.7	1882.2	1884.1	1.9
106	258	152	46	278	0.1783	63.0	1890.6	1892.7	2.2
106	259	153	47	278	0.1815	63.3	1898.8	1899.5	0.7
106	260	154	48	278	0.1846	63.7	1906.9	1907.9	1.0
106	261	155	49	278	0.1877	64.0	1915.0	1914.4	-0.5
106	262	156	50	278	0.1908	64.3	1922.9	1922.6	-0.3
106	263	157	51	278	0.1939	64.6	1930.7	1928.9	-1.8
106	264	158	52	278	0.1970	65.0	1938.4	1936.8	-1.6
106	265	159	53	278	0.2000	65.3	1946.0	1942.8	-3.1
106	266	160	54	278	0.2030	65.6	1953.5	1950.5	-2.9
106	267	161	55	278	0.2060	66.0	1960.8	1956.3	-4.5
106	268	162	56	278	0.2090	66.3	1968.1	1963.8	-4.3
106	269	163	57	278	0.2119	66.7	1975.3	1969.4	-5.9
106	270	164	58	278	0.2148	67.0	1982.3	1976.6	-5.7
106	271	165	59	278	0.2177	67.4	1989.3	1982.0	-7.3
106	272	166	60	278	0.2206	67.8	1996.2	1989.0	-7.1
106	273	167	61	278	0.2234	68.1	2002.9	1994.2	-8.7
106	274	168	62	278	0.2263	68.5	2009.6	2001.0	-8.6
106	275	169	63	278	0.2291	68.9	2016.1	2006.0	-10.1
106	276	170	64	278	0.2319	69.2	2022.6	2012.6	-10.0
106	277	171	65	278	0.2347	69.6	2028.9	2017.4	-11.6
106	278	172	66	278	0.2374	70.0	2035.2	2023.7	-11.5
106	279	173	67	278	0.2401	70.4	2041.3	2028.3	-13.0
106	280	174	68	278	0.2429	70.7	2047.4	2034.5	-12.9
106	281	175	69	278	0.2456	71.1	2053.3	2038.9	-14.5
106	282	176	70	278	0.2482	71.5	2059.2	2044.8	-14.4
106	283	177	71	278	0.2509	71.9	2064.9	2049.0	-15.9
106	284	178	72	278	0.2535	72.3	2070.6	2054.8	-15.8
106	285	179	73	278	0.2561	72.7	2076.1	2058.8	-17.4
106	286	180	74	278	0.2587	73.1	2081.6	2064.4	-17.3
106	287	181	75	278	0.2613	73.5	2087.0	2068.2	-18.8
106	288	182	76	278	0.2639	73.9	2092.2	2073.6	-18.7
106	289	183	77	278	0.2664	74.3	2097.4	2077.2	-20.2
106	290	184	78	278	0.2690	74.7	2102.5	2082.4	-20.1
106	291	185	79	278	0.2715	75.1	2107.5	2085.9	-21.6
106	292	186	80	278	0.2740	75.5	2112.4	2090.9	-21.5
106	293	187	81	278	0.2765	75.9	2117.2	2094.2	-23.0
106	294	188	82	278	0.2789	76.3	2121.9	2099.0	-22.9
106	295	189	83	278	0.2814	76.8	2126.5	2102.1	-24.4
106	296	190	84	278	0.2838	77.2	2131.0	2106.8	-24.2
106	297	191	85	278	0.2862	77.6	2135.4	2109.7	-25.7
106	298	192	86	278	0.2886	78.0	2139.7	2114.2	-25.5
106	299	193	87	278	0.2910	78.5	2144.0	2117.0	-27.0
106	300	194	88	278	0.2933	78.9	2148.1	2121.3	-26.8
106	301	195	89	278	0.2957	79.3	2152.2	2124.0	-28.2
106	302	196	90	278	0.2980	79.7	2156.1	2128.1	-28.0
106	303	197	91	278	0.3003	80.2	2160.0	2130.6	-29.4
106	304	198	92	278	0.3026	80.6	2163.8	2134.6	-29.2
106	305	199	93	278	0.3049	81.1	2167.4	2136.9	-30.6
106	306	200	94	278	0.3072	81.5	2171.0	2140.7	-30.3
106	307	201	95	278	0.3095	81.9	2174.5	2142.8	-31.7
106	308	202	96	278	0.3117	82.4	2177.9	2146.5	-31.4
106	309	203	97	278	0.3139	82.8	2181.3	2148.5	-32.7

106	310	204	98	278	0.3161	83.3	2184.5	2152.0	-32.5
106	311	205	99	278	0.3183	83.7	2187.6	2153.9	-33.8
106	312	206	100	278	0.3205	84.2	2190.7	2157.3	-33.4
106	313	207	101	278	0.3227	84.6	2193.6	2159.0	-34.7
106	314	208	102	278	0.3248	85.1	2196.5	2162.2	-34.4
106	315	209	103	278	0.3270	85.5	2199.3	2163.7	-35.6
106	316	210	104	278	0.3291	86.0	2202.0	2166.8	-35.2
106	317	211	105	278	0.3312	86.4	2204.6	2168.2	-36.4
106	318	212	106	278	0.3333	86.9	2207.1	2171.1	-36.0
106	319	213	107	278	0.3354	87.4	2209.6	2172.4	-37.1
106	320	214	108	278	0.3375	87.8	2211.9	2175.2	-36.7
106	321	215	109	278	0.3396	88.3	2214.2	2176.4	-37.8
106	322	216	110	278	0.3416	88.8	2216.3	2179.0	-37.3
106	323	217	111	278	0.3437	89.2	2218.4	2180.0	-38.4
106	324	218	112	278	0.3457	89.7	2220.4	2182.5	-37.9
106	325	219	113	278	0.3477	90.2	2222.3	2183.4	-38.9
106	326	220	114	278	0.3497	90.6	2224.1	2185.8	-38.4
106	327	221	115	278	0.3517	91.1	2225.8	2186.5	-39.3
106	328	222	116	278	0.3537	91.6	2227.5	2188.7	-38.8
106	329	223	117	278	0.3556	92.0	2229.0	2189.4	-39.7
106	330	224	118	278	0.3576	92.5	2230.5	2191.5	-39.1
106	331	225	119	278	0.3595	93.0	2231.9	2192.0	-40.0
106	332	226	120	278	0.3615	93.5	2233.2	2193.9	-39.3
106	333	227	121	278	0.3634	94.0	2234.4	2194.3	-40.1
106	334	228	122	278	0.3653	94.4	2235.5	2196.1	-39.4
106	335	229	123	278	0.3672	94.9	2236.6	2196.4	-40.2
106	336	230	124	278	0.3691	95.4	2237.5	2198.1	-39.4
106	337	231	125	278	0.3709	95.9	2238.4	2198.2	-40.2
106	338	232	126	278	0.3728	96.4	2239.2	2199.8	-39.4
106	339	233	127	278	0.3746	96.9	2239.9	2199.9	-40.1
106	340	234	128	278	0.3765	97.4	2240.5	2201.3	-39.2
106	341	235	129	278	0.3783	97.9	2241.1	2201.2	-39.8
106	342	236	130	278	0.3801	98.3	2241.5	2202.6	-38.9
106	343	237	131	278	0.3819	98.8	2241.9	2202.4	-39.5
106	344	238	132	278	0.3837	99.3	2242.2	2203.6	-38.5
106	345	239	133	278	0.3855	99.8	2242.3	2203.3	-39.1
106	346	240	134	278	0.3873	100.3	2242.5	2204.4	-38.1
106	347	241	135	278	0.3891	100.8	2242.5	2204.0	-38.5
106	348	242	136	278	0.3908	101.3	2242.4	2205.0	-37.5
106	349	243	137	278	0.3926	101.8	2242.3	2204.4	-37.9

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
111	218	107	-4	293	0.0184	59.7	1343.7	1351.0	7.3
111	219	108	-3	293	0.0137	59.7	1359.3	1367.7	8.4
111	220	109	-2	293	0.0091	59.7	1374.7	1382.6	7.9
111	221	110	-1	293	0.0045	59.7	1389.9	1398.9	9.0
111	222	111	0	293	0.0000	59.6	1404.9	1413.4	8.5
111	223	112	1	293	0.0045	59.7	1419.7	1429.3	9.6
111	224	113	2	293	0.0089	59.7	1434.3	1443.4	9.1
111	225	114	3	293	0.0133	59.7	1448.8	1458.9	10.1
111	226	115	4	293	0.0177	59.7	1463.0	1472.6	9.6
111	227	116	5	293	0.0220	59.8	1477.1	1487.7	10.6
111	228	117	6	293	0.0263	59.8	1491.0	1501.0	10.0
111	229	118	7	293	0.0306	59.9	1504.8	1515.8	11.0
111	230	119	8	293	0.0348	60.0	1518.3	1528.7	10.4
111	231	120	9	293	0.0390	60.1	1531.8	1543.1	11.3
111	232	121	10	293	0.0431	60.2	1545.0	1555.7	10.7

111	233	122	11	293	0.0472	60.3	1558.1	1569.7	11.6
111	234	123	12	293	0.0513	60.4	1571.0	1582.0	10.9
111	235	124	13	293	0.0553	60.5	1583.8	1595.6	11.8
111	236	125	14	293	0.0593	60.7	1596.4	1607.5	11.1
111	237	126	15	293	0.0633	60.8	1608.9	1620.8	11.9
111	238	127	16	293	0.0672	61.0	1621.2	1632.4	11.2
111	239	128	17	293	0.0711	61.1	1633.4	1645.4	12.0
111	240	129	18	293	0.0750	61.3	1645.4	1656.6	11.2
111	241	130	19	293	0.0788	61.5	1657.3	1669.2	11.9
111	242	131	20	293	0.0826	61.6	1669.1	1680.2	11.1
111	243	132	21	293	0.0864	61.8	1680.7	1692.5	11.8
111	244	133	22	293	0.0902	62.0	1692.2	1703.1	11.0
111	245	134	23	293	0.0939	62.2	1703.5	1715.1	11.6
111	246	135	24	293	0.0976	62.4	1714.7	1725.4	10.7
111	247	136	25	293	0.1012	62.6	1725.8	1737.1	11.3
111	248	137	26	293	0.1048	62.9	1736.7	1747.1	10.4
111	249	138	27	293	0.1084	63.1	1747.5	1758.5	11.0
111	250	139	28	293	0.1120	63.3	1758.2	1768.2	10.0
111	251	140	29	293	0.1155	63.6	1768.8	1779.3	10.5
111	252	141	30	293	0.1191	63.8	1779.2	1788.7	9.6
111	253	142	31	293	0.1225	64.0	1789.5	1799.5	10.0
111	254	143	32	293	0.1260	64.3	1799.7	1808.7	9.0
111	255	144	33	293	0.1294	64.6	1809.7	1819.2	9.5
111	256	145	34	293	0.1328	64.8	1819.7	1828.1	8.4
111	257	146	35	293	0.1362	65.1	1829.5	1838.3	8.8
111	258	147	36	293	0.1395	65.4	1839.2	1846.9	7.7
111	259	148	37	293	0.1429	65.6	1848.8	1856.9	8.1
111	260	149	38	293	0.1462	65.9	1858.2	1865.2	7.0
111	261	150	39	293	0.1494	66.2	1867.6	1874.9	7.3
111	262	151	40	293	0.1527	66.5	1876.8	1883.0	6.2
111	263	152	41	293	0.1559	66.8	1885.9	1892.4	6.5
111	264	153	42	293	0.1591	67.1	1895.0	1900.3	5.3
111	265	154	43	293	0.1623	67.4	1903.8	1909.5	5.6
111	266	155	44	293	0.1654	67.7	1912.6	1917.1	4.4
111	267	156	45	293	0.1685	68.0	1921.3	1926.0	4.7
111	268	157	46	293	0.1716	68.3	1929.9	1933.3	3.5
111	269	158	47	293	0.1747	68.7	1938.3	1942.0	3.7
111	270	159	48	293	0.1778	69.0	1946.7	1949.1	2.5
111	271	160	49	293	0.1808	69.3	1954.9	1957.5	2.6
111	272	161	50	293	0.1838	69.7	1963.0	1964.4	1.4
111	273	162	51	293	0.1868	70.0	1971.1	1972.6	1.6
111	274	163	52	293	0.1898	70.3	1979.0	1979.3	0.3
111	275	164	53	293	0.1927	70.7	1986.8	1987.3	0.5
111	276	165	54	293	0.1957	71.0	1994.5	1993.7	-0.8
111	277	166	55	293	0.1986	71.4	2002.1	2001.4	-0.7
111	278	167	56	293	0.2014	71.7	2009.6	2007.6	-2.0
111	279	168	57	293	0.2043	72.1	2017.0	2015.1	-1.9
111	280	169	58	293	0.2071	72.5	2024.3	2021.2	-3.2
111	281	170	59	293	0.2100	72.8	2031.5	2028.4	-3.1
111	282	171	60	293	0.2128	73.2	2038.6	2034.2	-4.4
111	283	172	61	293	0.2156	73.6	2045.6	2041.3	-4.3
111	284	173	62	293	0.2183	73.9	2052.5	2046.9	-5.6
111	285	174	63	293	0.2211	74.3	2059.3	2053.8	-5.5
111	286	175	64	293	0.2238	74.7	2066.0	2059.1	-6.9
111	287	176	65	293	0.2265	75.1	2072.6	2065.8	-6.8
111	288	177	66	293	0.2292	75.5	2079.1	2071.0	-8.1
111	289	178	67	293	0.2318	75.9	2085.5	2077.4	-8.1
111	290	179	68	293	0.2345	76.2	2091.8	2082.4	-9.4
111	291	180	69	293	0.2371	76.6	2098.0	2088.7	-9.3

111	292	181	70	293	0.2397	77.0	2104.1	2093.5	-10.7
111	293	182	71	293	0.2423	77.4	2110.2	2099.5	-10.6
111	294	183	72	293	0.2449	77.8	2116.1	2104.1	-11.9
111	295	184	73	293	0.2475	78.2	2121.9	2110.0	-11.9
111	296	185	74	293	0.2500	78.7	2127.6	2114.4	-13.2
111	297	186	75	293	0.2525	79.1	2133.3	2120.1	-13.2
111	298	187	76	293	0.2550	79.5	2138.8	2124.4	-14.5
111	299	188	77	293	0.2575	79.9	2144.3	2129.9	-14.4
111	300	189	78	293	0.2600	80.3	2149.6	2133.9	-15.7
111	301	190	79	293	0.2625	80.7	2154.9	2139.2	-15.7
111	302	191	80	293	0.2649	81.2	2160.1	2143.1	-17.0
111	303	192	81	293	0.2673	81.6	2165.2	2148.2	-16.9
111	304	193	82	293	0.2697	82.0	2170.2	2152.0	-18.2
111	305	194	83	293	0.2721	82.4	2175.1	2156.9	-18.1
111	306	195	84	293	0.2745	82.9	2179.9	2160.5	-19.4
111	307	196	85	293	0.2769	83.3	2184.6	2165.2	-19.3
111	308	197	86	293	0.2792	83.7	2189.2	2168.6	-20.6
111	309	198	87	293	0.2816	84.2	2193.8	2173.2	-20.5
111	310	199	88	293	0.2839	84.6	2198.2	2176.5	-21.7
111	311	200	89	293	0.2862	85.0	2202.6	2180.9	-21.7
111	312	201	90	293	0.2885	85.5	2206.8	2184.0	-22.9
111	313	202	91	293	0.2907	85.9	2211.0	2188.3	-22.8
111	314	203	92	293	0.2930	86.4	2215.1	2191.1	-24.0
111	315	204	93	293	0.2952	86.8	2219.1	2195.3	-23.8
111	316	205	94	293	0.2975	87.3	2223.0	2198.0	-25.0
111	317	206	95	293	0.2997	87.7	2226.8	2202.0	-24.9
111	318	207	96	293	0.3019	88.2	2230.6	2204.6	-26.0
111	319	208	97	293	0.3041	88.7	2234.2	2208.4	-25.9
111	320	209	98	293	0.3063	89.1	2237.8	2210.8	-27.0
111	321	210	99	293	0.3084	89.6	2241.3	2214.5	-26.8
111	322	211	100	293	0.3106	90.0	2244.7	2216.8	-27.9
111	323	212	101	293	0.3127	90.5	2248.0	2220.3	-27.7
111	324	213	102	293	0.3148	91.0	2251.2	2222.4	-28.8
111	325	214	103	293	0.3169	91.4	2254.3	2225.8	-28.5
111	326	215	104	293	0.3190	91.9	2257.4	2227.8	-29.6
111	327	216	105	293	0.3211	92.4	2260.3	2231.0	-29.3
111	328	217	106	293	0.3232	92.9	2263.2	2232.9	-30.3
111	329	218	107	293	0.3252	93.3	2266.0	2235.9	-30.0
111	330	219	108	293	0.3273	93.8	2268.7	2237.7	-31.0
111	331	220	109	293	0.3293	94.3	2271.3	2240.6	-30.7
111	332	221	110	293	0.3313	94.8	2273.8	2242.2	-31.6
111	333	222	111	293	0.3333	95.2	2276.3	2245.0	-31.3
111	334	223	112	293	0.3353	95.7	2278.6	2246.4	-32.2
111	335	224	113	293	0.3373	96.2	2280.9	2249.1	-31.8
111	336	225	114	293	0.3393	96.7	2283.1	2250.4	-32.7
111	337	226	115	293	0.3413	97.2	2285.2	2252.9	-32.3
111	338	227	116	293	0.3432	97.7	2287.3	2254.1	-33.1
111	339	228	117	293	0.3451	98.2	2289.2	2256.5	-32.7
111	340	229	118	293	0.3471	98.6	2291.1	2257.6	-33.5
111	341	230	119	293	0.3490	99.1	2292.8	2259.8	-33.0
111	342	231	120	293	0.3509	99.6	2294.5	2260.8	-33.7
111	343	232	121	293	0.3528	100.1	2296.1	2262.9	-33.2
111	344	233	122	293	0.3547	100.6	2297.7	2263.7	-33.9
111	345	234	123	293	0.3565	101.1	2299.1	2265.7	-33.4
111	346	235	124	293	0.3584	101.6	2300.5	2266.4	-34.0
111	347	236	125	293	0.3602	102.1	2301.8	2268.3	-33.4
111	348	237	126	293	0.3621	102.6	2303.0	2268.9	-34.1
111	349	238	127	293	0.3639	103.1	2304.1	2270.7	-33.4
111	350	239	128	293	0.3657	103.6	2305.1	2271.1	-34.0

111	351	240	129	293	0.3675	104.1	2306.1	2272.8	-33.3
111	352	241	130	293	0.3693	104.7	2306.9	2273.1	-33.8
111	353	242	131	293	0.3711	105.2	2307.7	2274.6	-33.1
111	354	243	132	293	0.3729	105.7	2308.4	2274.8	-33.6
111	355	244	133	293	0.3747	106.2	2309.1	2276.3	-32.8
111	356	245	134	293	0.3764	106.7	2309.6	2276.4	-33.2
111	357	246	135	293	0.3782	107.2	2310.1	2277.7	-32.4
111	358	247	136	293	0.3799	107.7	2310.5	2277.7	-32.8
111	359	248	137	293	0.3816	108.2	2310.8	2278.8	-31.9
111	360	249	138	293	0.3833	108.8	2311.0	2278.7	-32.3
111	361	250	139	293	0.3850	109.3	2311.1	2279.8	-31.3
111	362	251	140	293	0.3867	109.8	2311.2	2279.6	-31.6
111	363	252	141	293	0.3884	110.3	2311.2	2280.5	-30.6
111	364	253	142	293	0.3901	110.8	2311.1	2280.2	-30.9
111	365	254	143	293	0.3918	111.4	2310.9	2281.1	-29.8
111	366	255	144	293	0.3934	111.9	2310.6	2280.6	-30.0

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
116	228	112	-4	308	0.0175	65.2	1372.8	1381.2	8.4
116	229	113	-3	308	0.0131	65.1	1388.5	1396.2	7.7
116	230	114	-2	308	0.0087	65.1	1404.0	1412.9	9.0
116	231	115	-1	308	0.0043	65.1	1419.3	1427.6	8.3
116	232	116	0	308	0.0000	65.1	1434.3	1443.9	9.6
116	233	117	1	308	0.0043	65.1	1449.2	1458.1	8.9
116	234	118	2	308	0.0086	65.1	1464.0	1474.1	10.2
116	235	119	3	308	0.0128	65.1	1478.5	1488.0	9.5
116	236	120	4	308	0.0170	65.2	1492.9	1503.6	10.7
116	237	121	5	308	0.0211	65.2	1507.1	1517.1	10.0
116	238	122	6	308	0.0252	65.3	1521.1	1532.3	11.2
116	239	123	7	308	0.0293	65.4	1534.9	1545.4	10.5
116	240	124	8	308	0.0333	65.5	1548.6	1560.3	11.6
116	241	125	9	308	0.0373	65.5	1562.1	1573.0	10.9
116	242	126	10	308	0.0413	65.6	1575.5	1587.5	12.0
116	243	127	11	308	0.0453	65.7	1588.7	1600.0	11.3
116	244	128	12	308	0.0492	65.9	1601.8	1614.1	12.4
116	245	129	13	308	0.0531	66.0	1614.7	1626.2	11.6
116	246	130	14	308	0.0569	66.1	1627.4	1640.0	12.6
116	247	131	15	308	0.0607	66.3	1640.0	1651.8	11.8
116	248	132	16	308	0.0645	66.4	1652.5	1665.3	12.8
116	249	133	17	308	0.0683	66.6	1664.8	1676.8	12.0
116	250	134	18	308	0.0720	66.7	1676.9	1689.9	13.0
116	251	135	19	308	0.0757	66.9	1689.0	1701.1	12.1
116	252	136	20	308	0.0794	67.1	1700.9	1713.9	13.0
116	253	137	21	308	0.0830	67.3	1712.6	1724.8	12.1
116	254	138	22	308	0.0866	67.5	1724.2	1737.3	13.0
116	255	139	23	308	0.0902	67.7	1735.7	1747.8	12.1
116	256	140	24	308	0.0938	67.9	1747.1	1760.0	13.0
116	257	141	25	308	0.0973	68.1	1758.3	1770.3	12.0
116	258	142	26	308	0.1008	68.3	1769.4	1782.2	12.8
116	259	143	27	308	0.1043	68.6	1780.3	1792.2	11.8
116	260	144	28	308	0.1077	68.8	1791.2	1803.8	12.6
116	261	145	29	308	0.1111	69.0	1801.9	1813.5	11.6
116	262	146	30	308	0.1145	69.3	1812.5	1824.8	12.3
116	263	147	31	308	0.1179	69.5	1822.9	1834.2	11.3
116	264	148	32	308	0.1212	69.8	1833.3	1845.3	12.0
116	265	149	33	308	0.1245	70.1	1843.5	1854.4	10.9
116	266	150	34	308	0.1278	70.3	1853.6	1865.2	11.6

116	267	151	35	308	0.1311	70.6	1863.6	1874.1	10.5
116	268	152	36	308	0.1343	70.9	1873.5	1884.6	11.1
116	269	153	37	308	0.1376	71.2	1883.2	1893.2	10.0
116	270	154	38	308	0.1407	71.4	1892.8	1903.4	10.6
116	271	155	39	308	0.1439	71.7	1902.4	1911.8	9.4
116	272	156	40	308	0.1471	72.0	1911.8	1921.8	10.0
116	273	157	41	308	0.1502	72.3	1921.1	1929.9	8.8
116	274	158	42	308	0.1533	72.6	1930.3	1939.6	9.4
116	275	159	43	308	0.1564	73.0	1939.3	1947.5	8.2
116	276	160	44	308	0.1594	73.3	1948.3	1957.0	8.7
116	277	161	45	308	0.1625	73.6	1957.1	1964.6	7.4
116	278	162	46	308	0.1655	73.9	1965.9	1973.8	7.9
116	279	163	47	308	0.1685	74.2	1974.5	1981.2	6.7
116	280	164	48	308	0.1714	74.6	1983.1	1990.2	7.2
116	281	165	49	308	0.1744	74.9	1991.5	1997.4	5.9
116	282	166	50	308	0.1773	75.3	1999.8	2006.1	6.3
116	283	167	51	308	0.1802	75.6	2008.0	2013.1	5.0
116	284	168	52	308	0.1831	75.9	2016.1	2021.6	5.5
116	285	169	53	308	0.1860	76.3	2024.2	2028.3	4.2
116	286	170	54	308	0.1888	76.7	2032.1	2036.6	4.6
116	287	171	55	308	0.1916	77.0	2039.9	2043.1	3.2
116	288	172	56	308	0.1944	77.4	2047.6	2051.2	3.6
116	289	173	57	308	0.1972	77.7	2055.2	2057.5	2.3
116	290	174	58	308	0.2000	78.1	2062.7	2065.3	2.7
116	291	175	59	308	0.2028	78.5	2070.1	2071.4	1.3
116	292	176	60	308	0.2055	78.9	2077.4	2079.1	1.7
116	293	177	61	308	0.2082	79.2	2084.6	2084.9	0.3
116	294	178	62	308	0.2109	79.6	2091.7	2092.4	0.7
116	295	179	63	308	0.2136	80.0	2098.7	2098.0	-0.7
116	296	180	64	308	0.2162	80.4	2105.7	2105.3	-0.4
116	297	181	65	308	0.2189	80.8	2112.5	2110.7	-1.7
116	298	182	66	308	0.2215	81.2	2119.2	2117.8	-1.4
116	299	183	67	308	0.2241	81.6	2125.8	2123.1	-2.8
116	300	184	68	308	0.2267	82.0	2132.4	2129.9	-2.5
116	301	185	69	308	0.2292	82.4	2138.8	2135.0	-3.8
116	302	186	70	308	0.2318	82.8	2145.2	2141.6	-3.6
116	303	187	71	308	0.2343	83.2	2151.4	2146.5	-4.9
116	304	188	72	308	0.2368	83.6	2157.6	2152.9	-4.6
116	305	189	73	308	0.2393	84.0	2163.6	2157.6	-6.0
116	306	190	74	308	0.2418	84.4	2169.6	2163.9	-5.7
116	307	191	75	308	0.2443	84.9	2175.5	2168.4	-7.1
116	308	192	76	308	0.2468	85.3	2181.3	2174.5	-6.8
116	309	193	77	308	0.2492	85.7	2187.0	2178.8	-8.1
116	310	194	78	308	0.2516	86.1	2192.6	2184.7	-7.9
116	311	195	79	308	0.2540	86.6	2198.1	2188.9	-9.2
116	312	196	80	308	0.2564	87.0	2203.5	2194.6	-8.9
116	313	197	81	308	0.2588	87.4	2208.9	2198.6	-10.2
116	314	198	82	308	0.2612	87.9	2214.1	2204.2	-10.0
116	315	199	83	308	0.2635	88.3	2219.3	2208.0	-11.3
116	316	200	84	308	0.2658	88.8	2224.3	2213.3	-11.0
116	317	201	85	308	0.2681	89.2	2229.3	2217.0	-12.3
116	318	202	86	308	0.2704	89.6	2234.2	2222.2	-12.0
116	319	203	87	308	0.2727	90.1	2239.0	2225.7	-13.3
116	320	204	88	308	0.2750	90.5	2243.7	2230.7	-13.0
116	321	205	89	308	0.2773	91.0	2248.4	2234.1	-14.3
116	322	206	90	308	0.2795	91.5	2252.9	2238.9	-14.0
116	323	207	91	308	0.2817	91.9	2257.4	2242.1	-15.3
116	324	208	92	308	0.2840	92.4	2261.7	2246.8	-14.9
116	325	209	93	308	0.2862	92.8	2266.0	2249.8	-16.2

116	326	210	94	308	0.2883	93.3	2270.2	2254.4	-15.8
116	327	211	95	308	0.2905	93.8	2274.3	2257.2	-17.1
116	328	212	96	308	0.2927	94.2	2278.4	2261.6	-16.7
116	329	213	97	308	0.2948	94.7	2282.3	2264.4	-17.9
116	330	214	98	308	0.2970	95.2	2286.2	2268.6	-17.6
116	331	215	99	308	0.2991	95.6	2289.9	2271.2	-18.8
116	332	216	100	308	0.3012	96.1	2293.6	2275.2	-18.4
116	333	217	101	308	0.3033	96.6	2297.2	2277.7	-19.6
116	334	218	102	308	0.3054	97.1	2300.7	2281.6	-19.1
116	335	219	103	308	0.3075	97.6	2304.2	2283.9	-20.3
116	336	220	104	308	0.3095	98.0	2307.5	2287.7	-19.9
116	337	221	105	308	0.3116	98.5	2310.8	2289.8	-21.0
116	338	222	106	308	0.3136	99.0	2314.0	2293.4	-20.5
116	339	223	107	308	0.3156	99.5	2317.1	2295.5	-21.6
116	340	224	108	308	0.3177	100.0	2320.1	2298.9	-21.2
116	341	225	109	308	0.3197	100.5	2323.0	2300.8	-22.2
116	342	226	110	308	0.3216	101.0	2325.9	2304.2	-21.7
116	343	227	111	308	0.3236	101.5	2328.7	2305.9	-22.7
116	344	228	112	308	0.3256	102.0	2331.3	2309.1	-22.2
116	345	229	113	308	0.3275	102.5	2333.9	2310.7	-23.2
116	346	230	114	308	0.3295	103.0	2336.5	2313.8	-22.7
116	347	231	115	308	0.3314	103.5	2338.9	2315.3	-23.6
116	348	232	116	308	0.3333	104.0	2341.3	2318.2	-23.1
116	349	233	117	308	0.3352	104.5	2343.6	2319.6	-24.0
116	350	234	118	308	0.3371	105.0	2345.8	2322.4	-23.4
116	351	235	119	308	0.3390	105.5	2347.9	2323.6	-24.3
116	352	236	120	308	0.3409	106.0	2349.9	2326.3	-23.6
116	353	237	121	308	0.3428	106.5	2351.9	2327.4	-24.5
116	354	238	122	308	0.3446	107.0	2353.8	2329.9	-23.8
116	355	239	123	308	0.3465	107.5	2355.6	2330.9	-24.6
116	356	240	124	308	0.3483	108.0	2357.3	2333.4	-23.9
116	357	241	125	308	0.3501	108.6	2358.9	2334.2	-24.7
116	358	242	126	308	0.3520	109.1	2360.5	2336.5	-24.0
116	359	243	127	308	0.3538	109.6	2362.0	2337.3	-24.7
116	360	244	128	308	0.3556	110.1	2363.4	2339.4	-23.9
116	361	245	129	308	0.3573	110.6	2364.7	2340.1	-24.6
116	362	246	130	308	0.3591	111.2	2365.9	2342.1	-23.8
116	363	247	131	308	0.3609	111.7	2367.1	2342.6	-24.5
116	364	248	132	308	0.3626	112.2	2368.2	2344.6	-23.6
116	365	249	133	308	0.3644	112.7	2369.2	2345.0	-24.2
116	366	250	134	308	0.3661	113.3	2370.1	2346.8	-23.3
116	367	251	135	308	0.3679	113.8	2371.0	2347.1	-23.9
116	368	252	136	308	0.3696	114.3	2371.7	2348.8	-23.0
116	369	253	137	308	0.3713	114.9	2372.4	2349.0	-23.5
116	370	254	138	308	0.3730	115.4	2373.0	2350.6	-22.5
116	371	255	139	308	0.3747	115.9	2373.6	2350.6	-23.0
116	372	256	140	308	0.3763	116.5	2374.0	2352.1	-21.9
116	373	257	141	308	0.3780	117.0	2374.4	2352.1	-22.4
116	374	258	142	308	0.3797	117.5	2374.7	2353.4	-21.3
116	375	259	143	308	0.3813	118.1	2375.0	2353.3	-21.7
116	376	260	144	308	0.3830	118.6	2375.1	2354.6	-20.6
116	377	261	145	308	0.3846	119.2	2375.2	2354.3	-20.9
116	378	262	146	308	0.3862	119.7	2375.2	2355.5	-19.7
116	379	263	147	308	0.3879	120.2	2375.1	2355.1	-20.0
116	380	264	148	308	0.3895	120.8	2374.9	2356.2	-18.8
116	381	265	149	308	0.3911	121.3	2374.7	2355.7	-19.0
116	382	266	150	308	0.3927	121.9	2374.4	2356.7	-17.7

Proton number	Mass number	Neutron number	Excess neutron number	Est.stable mass number As	Beta value	Est.no of free nucleons	Estimated BE (MeV)	Reference BE (MeV)	Difference of BE (MeV)
121	238	117	-4	323	0.0168	70.9	1399.4	1404.9	5.5
121	239	118	-3	323	0.0126	70.8	1415.2	1421.7	6.6
121	240	119	-2	323	0.0083	70.8	1430.7	1436.8	6.1
121	241	120	-1	323	0.0042	70.8	1446.1	1453.3	7.2
121	242	121	0	323	0.0000	70.8	1461.3	1468.0	6.7
121	243	122	1	323	0.0041	70.8	1476.2	1484.0	7.8
121	244	123	2	323	0.0082	70.8	1491.1	1498.4	7.3
121	245	124	3	323	0.0122	70.8	1505.7	1514.1	8.4
121	246	125	4	323	0.0163	70.9	1520.1	1528.1	7.9
121	247	126	5	323	0.0202	70.9	1534.4	1543.4	9.0
121	248	127	6	323	0.0242	71.0	1548.5	1557.0	8.5
121	249	128	7	323	0.0281	71.1	1562.5	1572.0	9.5
121	250	129	8	323	0.0320	71.1	1576.3	1585.3	9.0
121	251	130	9	323	0.0359	71.2	1589.9	1600.0	10.0
121	252	131	10	323	0.0397	71.3	1603.4	1612.9	9.5
121	253	132	11	323	0.0435	71.4	1616.7	1627.2	10.5
121	254	133	12	323	0.0472	71.6	1629.9	1639.8	10.0
121	255	134	13	323	0.0510	71.7	1642.9	1653.8	10.9
121	256	135	14	323	0.0547	71.8	1655.8	1666.1	10.4
121	257	136	15	323	0.0584	72.0	1668.5	1679.7	11.3
121	258	137	16	323	0.0620	72.1	1681.0	1691.7	10.7
121	259	138	17	323	0.0656	72.3	1693.5	1705.1	11.6
121	260	139	18	323	0.0692	72.4	1705.8	1716.7	11.0
121	261	140	19	323	0.0728	72.6	1717.9	1729.7	11.8
121	262	141	20	323	0.0763	72.8	1729.9	1741.1	11.2
121	263	142	21	323	0.0799	73.0	1741.8	1753.8	12.0
121	264	143	22	323	0.0833	73.2	1753.6	1764.9	11.4
121	265	144	23	323	0.0868	73.4	1765.2	1777.3	12.2
121	266	145	24	323	0.0902	73.6	1776.7	1788.1	11.5
121	267	146	25	323	0.0936	73.8	1788.0	1800.2	12.2
121	268	147	26	323	0.0970	74.1	1799.2	1810.7	11.5
121	269	148	27	323	0.1004	74.3	1810.3	1822.5	12.2
121	270	149	28	323	0.1037	74.5	1821.3	1832.8	11.5
121	271	150	29	323	0.1070	74.8	1832.2	1844.3	12.2
121	272	151	30	323	0.1103	75.0	1842.9	1854.3	11.4
121	273	152	31	323	0.1136	75.3	1853.5	1865.5	12.1
121	274	153	32	323	0.1168	75.5	1864.0	1875.2	11.3
121	275	154	33	323	0.1200	75.8	1874.3	1886.2	11.9
121	276	155	34	323	0.1232	76.1	1884.6	1895.7	11.1
121	277	156	35	323	0.1264	76.3	1894.7	1906.4	11.7
121	278	157	36	323	0.1295	76.6	1904.7	1915.6	10.8
121	279	158	37	323	0.1326	76.9	1914.6	1926.0	11.4
121	280	159	38	323	0.1357	77.2	1924.4	1934.9	10.5
121	281	160	39	323	0.1388	77.5	1934.1	1945.2	11.1
121	282	161	40	323	0.1418	77.8	1943.7	1953.8	10.2
121	283	162	41	323	0.1449	78.1	1953.1	1963.8	10.7
121	284	163	42	323	0.1479	78.4	1962.5	1972.2	9.8
121	285	164	43	323	0.1509	78.7	1971.7	1981.9	10.2
121	286	165	44	323	0.1539	79.1	1980.8	1990.1	9.3
121	287	166	45	323	0.1568	79.4	1989.8	1999.6	9.8
121	288	167	46	323	0.1597	79.7	1998.8	2007.5	8.8
121	289	168	47	323	0.1626	80.0	2007.6	2016.8	9.2
121	290	169	48	323	0.1655	80.4	2016.3	2024.5	8.3
121	291	170	49	323	0.1684	80.7	2024.9	2033.5	8.7
121	292	171	50	323	0.1712	81.1	2033.4	2041.0	7.7

121	293	172	51	323	0.1741	81.4	2041.7	2049.8	8.1
121	294	173	52	323	0.1769	81.8	2050.0	2057.1	7.0
121	295	174	53	323	0.1797	82.1	2058.2	2065.6	7.4
121	296	175	54	323	0.1824	82.5	2066.3	2072.7	6.4
121	297	176	55	323	0.1852	82.9	2074.3	2081.0	6.7
121	298	177	56	323	0.1879	83.2	2082.2	2087.9	5.7
121	299	178	57	323	0.1906	83.6	2090.0	2096.0	6.0
121	300	179	58	323	0.1933	84.0	2097.7	2102.6	5.0
121	301	180	59	323	0.1960	84.4	2105.3	2110.5	5.3
121	302	181	60	323	0.1987	84.7	2112.7	2117.0	4.2
121	303	182	61	323	0.2013	85.1	2120.1	2124.7	4.5
121	304	183	62	323	0.2040	85.5	2127.4	2130.9	3.5
121	305	184	63	323	0.2066	85.9	2134.7	2138.4	3.8
121	306	185	64	323	0.2092	86.3	2141.8	2144.4	2.7
121	307	186	65	323	0.2117	86.7	2148.8	2151.7	3.0
121	308	187	66	323	0.2143	87.1	2155.7	2157.6	1.9
121	309	188	67	323	0.2168	87.5	2162.5	2164.7	2.1
121	310	189	68	323	0.2194	87.9	2169.3	2170.3	1.0
121	311	190	69	323	0.2219	88.3	2175.9	2177.2	1.3
121	312	191	70	323	0.2244	88.8	2182.5	2182.7	0.2
121	313	192	71	323	0.2268	89.2	2188.9	2189.4	0.5
121	314	193	72	323	0.2293	89.6	2195.3	2194.6	-0.6
121	315	194	73	323	0.2318	90.0	2201.6	2201.2	-0.4
121	316	195	74	323	0.2342	90.5	2207.7	2206.3	-1.5
121	317	196	75	323	0.2366	90.9	2213.8	2212.6	-1.2
121	318	197	76	323	0.2390	91.3	2219.8	2217.5	-2.3
121	319	198	77	323	0.2414	91.8	2225.8	2223.7	-2.1
121	320	199	78	323	0.2438	92.2	2231.6	2228.4	-3.2
121	321	200	79	323	0.2461	92.6	2237.3	2234.4	-2.9
121	322	201	80	323	0.2485	93.1	2243.0	2239.0	-4.0
121	323	202	81	323	0.2508	93.5	2248.5	2244.8	-3.8
121	324	203	82	323	0.2531	94.0	2254.0	2249.1	-4.9
121	325	204	83	323	0.2554	94.4	2259.4	2254.8	-4.6
121	326	205	84	323	0.2577	94.9	2264.7	2259.0	-5.7
121	327	206	85	323	0.2599	95.3	2269.9	2264.5	-5.5
121	328	207	86	323	0.2622	95.8	2275.0	2268.5	-6.5
121	329	208	87	323	0.2644	96.2	2280.1	2273.8	-6.3
121	330	209	88	323	0.2667	96.7	2285.0	2277.7	-7.3
121	331	210	89	323	0.2689	97.2	2289.9	2282.9	-7.1
121	332	211	90	323	0.2711	97.6	2294.7	2286.6	-8.1
121	333	212	91	323	0.2733	98.1	2299.4	2291.6	-7.8
121	334	213	92	323	0.2755	98.6	2304.0	2295.2	-8.9
121	335	214	93	323	0.2776	99.0	2308.5	2300.0	-8.6
121	336	215	94	323	0.2798	99.5	2313.0	2303.4	-9.6
121	337	216	95	323	0.2819	100.0	2317.4	2308.0	-9.3
121	338	217	96	323	0.2840	100.5	2321.6	2311.3	-10.3
121	339	218	97	323	0.2861	101.0	2325.8	2315.8	-10.0
121	340	219	98	323	0.2882	101.4	2329.9	2319.0	-11.0
121	341	220	99	323	0.2903	101.9	2334.0	2323.3	-10.7
121	342	221	100	323	0.2924	102.4	2337.9	2326.3	-11.6
121	343	222	101	323	0.2945	102.9	2341.8	2330.5	-11.3
121	344	223	102	323	0.2965	103.4	2345.6	2333.3	-12.2
121	345	224	103	323	0.2986	103.9	2349.3	2337.4	-11.9
121	346	225	104	323	0.3006	104.4	2352.9	2340.1	-12.8
121	347	226	105	323	0.3026	104.9	2356.4	2344.0	-12.5
121	348	227	106	323	0.3046	105.4	2359.9	2346.5	-13.4
121	349	228	107	323	0.3066	105.9	2363.3	2350.3	-13.0
121	350	229	108	323	0.3086	106.4	2366.6	2352.7	-13.8
121	351	230	109	323	0.3105	106.9	2369.8	2356.4	-13.4

121	352	231	110	323	0.3125	107.4	2372.9	2358.6	-14.3
121	353	232	111	323	0.3145	107.9	2376.0	2362.1	-13.9
121	354	233	112	323	0.3164	108.4	2379.0	2364.3	-14.7
121	355	234	113	323	0.3183	108.9	2381.9	2367.6	-14.2
121	356	235	114	323	0.3202	109.4	2384.7	2369.6	-15.0
121	357	236	115	323	0.3221	110.0	2387.4	2372.9	-14.6
121	358	237	116	323	0.3240	110.5	2390.1	2374.7	-15.3
121	359	238	117	323	0.3259	111.0	2392.7	2377.8	-14.8
121	360	239	118	323	0.3278	111.5	2395.2	2379.6	-15.6
121	361	240	119	323	0.3296	112.0	2397.6	2382.5	-15.0
121	362	241	120	323	0.3315	112.6	2399.9	2384.2	-15.7
121	363	242	121	323	0.3333	113.1	2402.2	2387.0	-15.2
121	364	243	122	323	0.3352	113.6	2404.4	2388.5	-15.9
121	365	244	123	323	0.3370	114.1	2406.5	2391.2	-15.3
121	366	245	124	323	0.3388	114.7	2408.5	2392.6	-15.9
121	367	246	125	323	0.3406	115.2	2410.5	2395.2	-15.3
121	368	247	126	323	0.3424	115.7	2412.4	2396.5	-15.9
121	369	248	127	323	0.3442	116.3	2414.2	2398.9	-15.2
121	370	249	128	323	0.3460	116.8	2415.9	2400.1	-15.8
121	371	250	129	323	0.3477	117.3	2417.5	2402.4	-15.1
121	372	251	130	323	0.3495	117.9	2419.1	2403.4	-15.7
121	373	252	131	323	0.3512	118.4	2420.6	2405.6	-15.0
121	374	253	132	323	0.3529	118.9	2422.0	2406.6	-15.4
121	375	254	133	323	0.3547	119.5	2423.3	2408.7	-14.7
121	376	255	134	323	0.3564	120.0	2424.6	2409.5	-15.2
121	377	256	135	323	0.3581	120.6	2425.8	2411.4	-14.4
121	378	257	136	323	0.3598	121.1	2426.9	2412.1	-14.8
121	379	258	137	323	0.3615	121.7	2427.9	2414.0	-13.9
121	380	259	138	323	0.3632	122.2	2428.9	2414.6	-14.3
121	381	260	139	323	0.3648	122.8	2429.8	2416.3	-13.5
121	382	261	140	323	0.3665	123.3	2430.6	2416.8	-13.8
121	383	262	141	323	0.3682	123.9	2431.3	2418.5	-12.9
121	384	263	142	323	0.3698	124.4	2432.0	2418.8	-13.2
121	385	264	143	323	0.3714	125.0	2432.6	2420.4	-12.2
121	386	265	144	323	0.3731	125.5	2433.1	2420.6	-12.5
121	387	266	145	323	0.3747	126.1	2433.5	2422.0	-11.5
121	388	267	146	323	0.3763	126.7	2433.9	2422.2	-11.7
121	389	268	147	323	0.3779	127.2	2434.2	2423.5	-10.6
121	390	269	148	323	0.3795	127.8	2434.4	2423.6	-10.8
121	391	270	149	323	0.3811	128.3	2434.5	2424.8	-9.7
121	392	271	150	323	0.3827	128.9	2434.6	2424.7	-9.8
121	393	272	151	323	0.3842	129.5	2434.6	2425.9	-8.7
121	394	273	152	323	0.3858	130.0	2434.5	2425.7	-8.8
121	395	274	153	323	0.3873	130.6	2434.3	2426.7	-7.6
121	396	275	154	323	0.3889	131.2	2434.1	2426.5	-7.6
121	397	276	155	323	0.3904	131.7	2433.8	2427.4	-6.4
121	398	277	156	323	0.3920	132.3	2433.4	2427.0	-6.4
121	399	278	157	323	0.3935	132.9	2432.9	2427.8	-5.1

Based on this relation (9), in a trial-error approach, we have developed another relation for estimating the maximum binding energy associated with any mass number. Very interesting point is that, it is independent of proton number, it can be expressed as [1,2],

$$BE \cong \left\{ A - \left(\sqrt{\frac{e}{e_n}} \right) 0.001605 A^2 - A^{1/3} - A^{-1/2} \right\} 10.1 \text{ MeV}$$
$$\cong \left\{ A - 0.000935 A^2 - A^{1/3} - A^{-1/2} \right\} 10.1 \text{ MeV} \quad (10)$$

Considering isobars and finding the maximum binding energy associated with each mass number, above relation can be verified. See the following Table.3 and Figure 1. Our proposal is failing for A=4, A= 202 to 212 and A >267. It needs a review with respect to shell effects and other microscopic corrections.

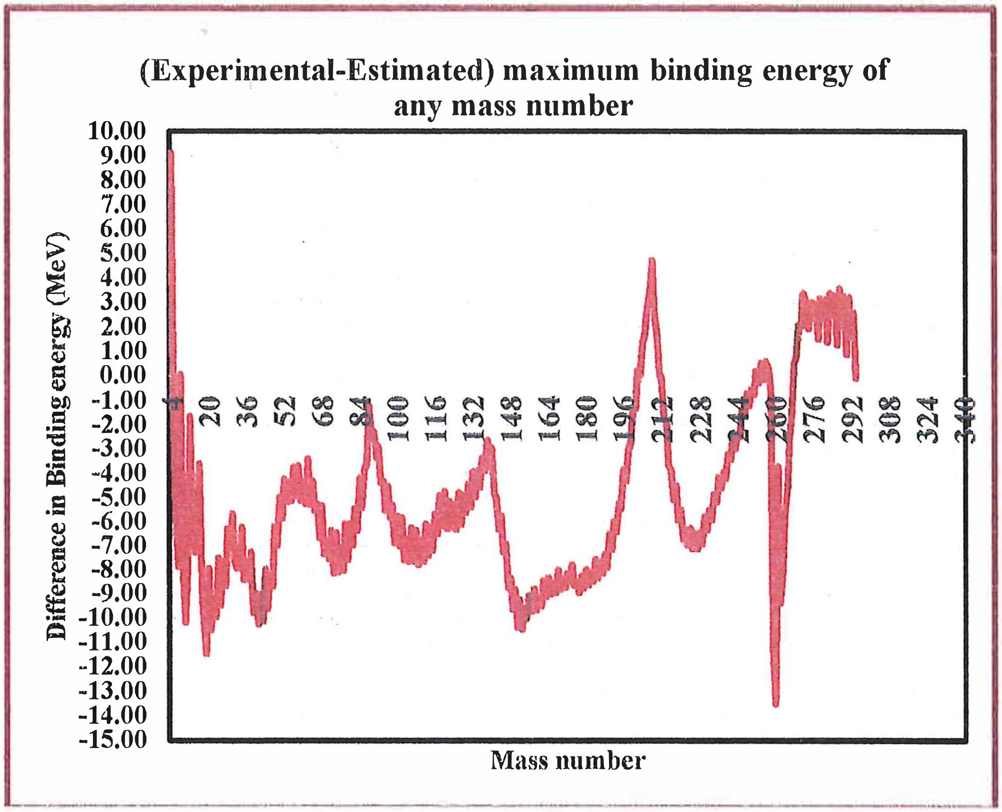


Figure 1: Difference of experimental and estimated maximum binding energy of A

Table.3 Estimated maximum binding energy of any mass number					
Assumed mass number A	Estimated Max. Binding energy of A (MeV)	Estimated Max. Binding energy per nucleon (MeV)	Experimental Max. Binding energy of A (MeV)	Experimental Max. Binding energy per nucleon (MeV)	(Experimental -Estimated) Binding energy (MeV)
4	19.17	4.79	28.3	7.08	9.13
5	28.48	5.70	27.56	5.51	-0.92
6	37.78	6.30	31.99	5.33	-5.79
7	47.1	6.73	39.25	5.61	-7.85
8	56.42	7.05	56.5	7.06	0.08
9	65.76	7.31	58.16	6.46	-7.6
10	75.1	7.51	64.98	6.50	-10.12
11	84.45	7.68	76.2	6.93	-8.25
12	93.8	7.82	92.16	7.68	-1.64
13	103.15	7.93	97.11	7.47	-6.04
14	112.51	8.04	105.28	7.52	-7.23
15	121.86	8.12	115.49	7.70	-6.37
16	131.21	8.20	127.62	7.98	-3.59
17	140.55	8.27	131.76	7.75	-8.79
18	149.89	8.33	139.81	7.77	-10.08
19	159.22	8.38	147.8	7.78	-11.42
20	168.55	8.43	160.64	8.03	-7.91
21	177.87	8.47	167.41	7.97	-10.46
22	187.18	8.51	177.77	8.08	-9.41
23	196.48	8.54	186.56	8.11	-9.92
24	205.77	8.57	198.26	8.26	-7.51
25	215.05	8.60	205.59	8.22	-9.46
26	224.31	8.63	216.68	8.33	-7.63
27	233.57	8.65	224.95	8.33	-8.62
28	242.82	8.67	236.54	8.45	-6.28
29	252.05	8.69	245.01	8.45	-7.04
30	261.27	8.71	255.62	8.52	-5.65
31	270.48	8.73	262.92	8.48	-7.56
32	279.68	8.74	271.78	8.49	-7.9
33	288.86	8.75	280.96	8.51	-7.9
34	298.03	8.77	291.84	8.58	-6.19
35	307.19	8.78	298.82	8.54	-8.37
36	316.33	8.79	308.71	8.58	-7.62
37	325.46	8.80	317.1	8.57	-8.36
38	334.57	8.80	327.34	8.61	-7.23
39	343.67	8.81	333.94	8.56	-9.73
40	352.75	8.82	343.81	8.60	-8.94
41	361.82	8.82	351.62	8.58	-10.2

42	370.88	8.83	361.9	8.62	-8.98
43	379.91	8.84	369.83	8.60	-10.08
44	388.94	8.84	380.96	8.66	-7.98
45	397.95	8.84	388.37	8.63	-9.58
46	406.94	8.85	398.77	8.67	-8.17
47	415.92	8.85	407.26	8.67	-8.66
48	424.88	8.85	418.7	8.72	-6.18
49	433.82	8.85	426.85	8.71	-6.97
50	442.75	8.86	437.78	8.76	-4.97
51	451.67	8.86	445.85	8.74	-5.82
52	460.57	8.86	456.35	8.78	-4.22
53	469.45	8.86	464.29	8.76	-5.16
54	478.31	8.86	474.01	8.78	-4.3
55	487.16	8.86	482.08	8.77	-5.08
56	495.99	8.86	492.26	8.79	-3.73
57	504.81	8.86	499.91	8.77	-4.9
58	513.61	8.86	509.95	8.79	-3.66
59	522.39	8.85	517.31	8.77	-5.08
60	531.16	8.85	526.85	8.78	-4.31
61	539.91	8.85	534.67	8.77	-5.24
62	548.64	8.85	545.26	8.79	-3.38
63	557.36	8.85	552.1	8.76	-5.26
64	566.06	8.84	561.76	8.78	-4.3
65	574.74	8.84	569.21	8.76	-5.53
66	583.4	8.84	578.14	8.76	-5.26
67	592.05	8.84	585.41	8.74	-6.64
68	600.68	8.83	595.39	8.76	-5.29
69	609.3	8.83	602	8.72	-7.3
70	617.89	8.83	611.09	8.73	-6.8
71	626.47	8.82	618.95	8.72	-7.52
72	635.04	8.82	628.69	8.73	-6.35
73	643.58	8.82	635.47	8.71	-8.11
74	652.11	8.81	645.66	8.73	-6.45
75	660.62	8.81	652.57	8.70	-8.05
76	669.11	8.80	662.07	8.71	-7.04
77	677.59	8.80	669.59	8.70	-8
78	686.05	8.80	679.99	8.72	-6.06
79	694.49	8.79	686.95	8.70	-7.54
80	702.91	8.79	696.87	8.71	-6.04
81	711.32	8.78	704.37	8.70	-6.95
82	719.71	8.78	714.27	8.71	-5.44
83	728.08	8.77	721.74	8.70	-6.34
84	736.43	8.77	732.27	8.72	-4.16
85	744.77	8.76	739.38	8.70	-5.39
86	753.09	8.76	749.23	8.71	-3.86
87	761.39	8.75	757.86	8.71	-3.53
88	769.67	8.75	768.47	8.73	-1.2

89	777.93	8.74	775.54	8.71	-2.39
90	786.18	8.74	783.9	8.71	-2.28
91	794.41	8.73	791.09	8.69	-3.32
92	802.62	8.72	799.73	8.69	-2.89
93	810.82	8.72	806.46	8.67	-4.36
94	818.99	8.71	814.68	8.67	-4.31
95	827.15	8.71	821.63	8.65	-5.52
96	835.29	8.70	830.78	8.65	-4.51
97	843.41	8.69	837.6	8.64	-5.81
98	851.52	8.69	846.25	8.64	-5.27
99	859.61	8.68	852.75	8.61	-6.86
100	867.67	8.68	861.93	8.62	-5.74
101	875.73	8.67	868.73	8.60	-7
102	883.76	8.66	877.95	8.61	-5.81
103	891.77	8.66	884.19	8.58	-7.58
104	899.77	8.65	893.09	8.59	-6.68
105	907.75	8.65	900.13	8.57	-7.62
106	915.71	8.64	909.48	8.58	-6.23
107	923.66	8.63	916.02	8.56	-7.64
108	931.58	8.63	925.24	8.57	-6.34
109	939.49	8.62	931.72	8.55	-7.77
110	947.38	8.61	940.64	8.55	-6.74
111	955.25	8.61	947.62	8.54	-7.63
112	963.1	8.60	957.01	8.54	-6.09
113	970.94	8.59	963.55	8.53	-7.39
114	978.75	8.59	972.59	8.53	-6.16
115	986.55	8.58	979.4	8.52	-7.15
116	994.33	8.57	988.68	8.52	-5.65
117	1002.1	8.56	995.62	8.51	-6.48
118	1009.84	8.56	1004.95	8.52	-4.89
119	1017.57	8.55	1011.43	8.50	-6.14
120	1025.27	8.54	1020.54	8.50	-4.73
121	1032.96	8.54	1026.71	8.49	-6.25
122	1040.64	8.53	1035.52	8.49	-5.12
123	1048.29	8.52	1042.1	8.47	-6.19
124	1055.92	8.52	1050.69	8.47	-5.23
125	1063.54	8.51	1057.27	8.46	-6.27
126	1071.14	8.50	1066.37	8.46	-4.77
127	1078.72	8.49	1072.66	8.45	-6.06
128	1086.28	8.49	1081.44	8.45	-4.84
129	1093.83	8.48	1088.24	8.44	-5.59
130	1101.35	8.47	1096.91	8.44	-4.44
131	1108.86	8.46	1103.51	8.42	-5.35
132	1116.35	8.46	1112.45	8.43	-3.9
133	1123.82	8.45	1118.88	8.41	-4.94
134	1131.28	8.44	1127.43	8.41	-3.85
135	1138.71	8.43	1134.18	8.40	-4.53

136	1146.13	8.43	1142.77	8.40	-3.36
137	1153.53	8.42	1149.68	8.39	-3.85
138	1160.9	8.41	1158.29	8.39	-2.61
139	1168.27	8.40	1164.55	8.38	-3.72
140	1175.61	8.40	1172.69	8.38	-2.92
141	1182.93	8.39	1178.12	8.36	-4.81
142	1190.24	8.38	1185.28	8.35	-4.96
143	1197.53	8.37	1191.26	8.33	-6.27
144	1204.8	8.37	1199.08	8.33	-5.72
145	1212.05	8.36	1204.83	8.31	-7.22
146	1219.28	8.35	1212.4	8.30	-6.88
147	1226.5	8.34	1217.8	8.28	-8.7
148	1233.69	8.34	1225.39	8.28	-8.3
149	1240.87	8.33	1231.26	8.26	-9.61
150	1248.03	8.32	1239.24	8.26	-8.79
151	1255.17	8.31	1244.84	8.24	-10.33
152	1262.3	8.30	1253.1	8.24	-9.2
153	1269.4	8.30	1258.99	8.23	-10.41
154	1276.49	8.29	1266.93	8.23	-9.56
155	1283.55	8.28	1273.58	8.22	-9.97
156	1290.6	8.27	1281.59	8.22	-9.01
157	1297.63	8.27	1287.95	8.20	-9.68
158	1304.65	8.26	1295.89	8.20	-8.76
159	1311.64	8.25	1302.02	8.19	-9.62
160	1318.62	8.24	1309.45	8.18	-9.17
161	1325.57	8.23	1316.09	8.17	-9.48
162	1332.51	8.23	1324.1	8.17	-8.41
163	1339.43	8.22	1330.37	8.16	-9.06
164	1346.33	8.21	1338.03	8.16	-8.3
165	1353.22	8.20	1344.25	8.15	-8.97
166	1360.08	8.19	1351.56	8.14	-8.52
167	1366.93	8.19	1358	8.13	-8.93
168	1373.76	8.18	1365.77	8.13	-7.99
169	1380.57	8.17	1371.78	8.12	-8.79
170	1387.36	8.16	1379.03	8.11	-8.33
171	1394.13	8.15	1385.42	8.10	-8.71
172	1400.88	8.14	1392.76	8.10	-8.12
173	1407.62	8.14	1399.13	8.09	-8.49
174	1414.34	8.13	1406.59	8.08	-7.75
175	1421.04	8.12	1412.41	8.07	-8.63
176	1427.72	8.11	1419.28	8.06	-8.44
177	1434.38	8.10	1425.46	8.05	-8.92
178	1441.02	8.10	1432.8	8.05	-8.22
179	1447.64	8.09	1438.9	8.04	-8.74
180	1454.25	8.08	1446.29	8.03	-7.96
181	1460.84	8.07	1452.24	8.02	-8.6
182	1467.41	8.06	1459.33	8.02	-8.08

183	1473.96	8.05	1465.52	8.01	-8.44
184	1480.49	8.05	1472.94	8.01	-7.55
185	1487	8.04	1478.69	7.99	-8.31
186	1493.5	8.03	1485.88	7.99	-7.62
187	1499.98	8.02	1491.88	7.98	-8.1
188	1506.43	8.01	1499.09	7.97	-7.34
189	1512.87	8.00	1505.01	7.96	-7.86
190	1519.29	8.00	1512.8	7.96	-6.49
191	1525.7	7.99	1518.56	7.95	-7.14
192	1532.08	7.98	1526.12	7.95	-5.96
193	1538.44	7.97	1532.06	7.94	-6.38
194	1544.79	7.96	1539.58	7.94	-5.21
195	1551.12	7.95	1545.68	7.93	-5.44
196	1557.43	7.95	1553.6	7.93	-3.83
197	1563.72	7.94	1559.45	7.92	-4.27
198	1569.99	7.93	1567	7.91	-2.99
199	1576.25	7.92	1573.48	7.91	-2.77
200	1582.48	7.91	1581.18	7.91	-1.3
201	1588.7	7.90	1587.41	7.90	-1.29
202	1594.9	7.90	1595.16	7.90	0.26
203	1601.07	7.89	1601.16	7.89	0.09
204	1607.24	7.88	1608.65	7.89	1.41
205	1613.38	7.87	1615.07	7.88	1.69
206	1619.5	7.86	1622.32	7.88	2.82
207	1625.61	7.85	1629.06	7.87	3.45
208	1631.69	7.84	1636.43	7.87	4.74
209	1637.76	7.84	1640.37	7.85	2.61
210	1643.81	7.83	1645.55	7.84	1.74
211	1649.84	7.82	1649.97	7.82	0.13
212	1655.85	7.81	1655.77	7.81	-0.08
213	1661.85	7.80	1660.13	7.79	-1.72
214	1667.82	7.79	1666.01	7.79	-1.81
215	1673.78	7.79	1670.16	7.77	-3.62
216	1679.72	7.78	1675.9	7.76	-3.82
217	1685.64	7.77	1680.58	7.74	-5.06
218	1691.54	7.76	1687.05	7.74	-4.49
219	1697.42	7.75	1691.51	7.72	-5.91
220	1703.28	7.74	1697.79	7.72	-5.49
221	1709.13	7.73	1702.42	7.70	-6.71
222	1714.95	7.73	1708.66	7.70	-6.29
223	1720.76	7.72	1713.82	7.69	-6.94
224	1726.55	7.71	1720.3	7.68	-6.25
225	1732.32	7.70	1725.21	7.67	-7.11
226	1738.07	7.69	1731.6	7.66	-6.47
227	1743.8	7.68	1736.71	7.65	-7.09
228	1749.52	7.67	1743.08	7.65	-6.44
229	1755.21	7.66	1748.33	7.63	-6.88

230	1760.89	7.66	1755.13	7.63	-5.76
231	1766.55	7.65	1760.25	7.62	-6.3
232	1772.19	7.64	1766.69	7.62	-5.5
233	1777.81	7.63	1771.93	7.60	-5.88
234	1783.41	7.62	1778.57	7.60	-4.84
235	1789	7.61	1783.86	7.59	-5.14
236	1794.56	7.60	1790.41	7.59	-4.15
237	1800.11	7.60	1795.53	7.58	-4.58
238	1805.64	7.59	1801.69	7.57	-3.95
239	1811.15	7.58	1806.97	7.56	-4.18
240	1816.64	7.57	1813.45	7.56	-3.19
241	1822.11	7.56	1818.69	7.55	-3.42
242	1827.56	7.55	1825	7.54	-2.56
243	1833	7.54	1830.03	7.53	-2.97
244	1838.41	7.53	1836.05	7.52	-2.36
245	1843.81	7.53	1841.36	7.52	-2.45
246	1849.19	7.52	1847.82	7.51	-1.37
247	1854.55	7.51	1852.98	7.50	-1.57
248	1859.89	7.50	1859.19	7.50	-0.7
249	1865.21	7.49	1864.02	7.49	-1.19
250	1870.52	7.48	1869.99	7.48	-0.53
251	1875.8	7.47	1875.09	7.47	-0.71
252	1881.07	7.46	1881.27	7.47	0.2
253	1886.32	7.46	1886.07	7.45	-0.25
254	1891.55	7.45	1892.1	7.45	0.55
255	1896.76	7.44	1896.64	7.44	-0.12
256	1901.95	7.43	1902.54	7.43	0.59
257	1907.12	7.42	1907.5	7.42	0.38
258	1912.28	7.41	1911.69	7.41	-0.59
259	1917.41	7.40	1906.33	7.36	-11.08
260	1922.53	7.39	1909.07	7.34	-13.46
261	1927.63	7.39	1923.93	7.37	-3.7
262	1932.71	7.38	1923.39	7.34	-9.32
263	1937.77	7.37	1929.63	7.34	-8.14
264	1942.81	7.36	1937.23	7.34	-5.58
265	1947.84	7.35	1943.25	7.33	-4.59
266	1952.84	7.34	1950.31	7.33	-2.53
267	1957.83	7.33	1956.31	7.33	-1.52
268	1962.8	7.32	1963.37	7.33	0.57
269	1967.74	7.32	1968.54	7.32	0.8
270	1972.67	7.31	1974.78	7.31	2.11
271	1977.59	7.30	1979.66	7.31	2.07
272	1982.48	7.29	1985.87	7.30	3.39
273	1987.35	7.28	1990.44	7.29	3.09
274	1992.21	7.27	1994.17	7.28	1.96
275	1997.05	7.26	2000.08	7.27	3.03
276	2001.86	7.25	2004.86	7.26	3

277	2006.66	7.24	2009.64	7.26	2.98
278	2011.44	7.24	2013	7.24	1.56
279	2016.21	7.23	2019.4	7.24	3.19
280	2020.95	7.22	2023.56	7.23	2.61
281	2025.68	7.21	2028.82	7.22	3.14
282	2030.38	7.20	2031.81	7.21	1.43
283	2035.07	7.19	2038.45	7.20	3.38
284	2039.74	7.18	2042.53	7.19	2.79
285	2044.39	7.17	2047.73	7.19	3.34
286	2049.02	7.16	2050.33	7.17	1.31
287	2053.63	7.16	2057.22	7.17	3.59
288	2058.22	7.15	2060.64	7.16	2.42
289	2062.8	7.14	2066.06	7.15	3.26
290	2067.36	7.13	2068.28	7.13	0.92
291	2071.89	7.12	2075.12	7.13	3.23
292	2076.41	7.11	2078.16	7.12	1.75
293	2080.91	7.10	2083.52	7.11	2.61
294	2085.39	7.09	2085.34	7.09	-0.05
295	2089.86	7.08			
296	2094.3	7.08			
297	2098.73	7.07			
298	2103.13	7.06			
299	2107.52	7.05			
300	2111.89	7.04			
301	2116.24	7.03			
302	2120.57	7.02			
303	2124.88	7.01			
304	2129.18	7.00			
305	2133.45	6.99			
306	2137.71	6.99			
307	2141.95	6.98			
308	2146.17	6.97			
309	2150.37	6.96			
310	2154.55	6.95			
311	2158.71	6.94			
312	2162.86	6.93			
313	2166.98	6.92			
314	2171.09	6.91			
315	2175.18	6.91			
316	2179.25	6.90			
317	2183.3	6.89			
318	2187.33	6.88			
319	2191.34	6.87			
320	2195.34	6.86			
321	2199.31	6.85			
322	2203.27	6.84			
323	2207.21	6.83			

324	2211.13	6.82			
325	2215.03	6.82			
326	2218.91	6.81			
327	2222.77	6.80			
328	2226.62	6.79			
329	2230.44	6.78			
330	2234.25	6.77			
331	2238.04	6.76			
332	2241.81	6.75			
333	2245.56	6.74			
334	2249.29	6.73			
335	2253.01	6.73			
336	2256.7	6.72			
337	2260.38	6.71			
338	2264.03	6.70			
339	2267.67	6.69			
340	2271.29	6.68			

Close to the maximum binding energy of any mass number, number of free nucleons can be expressed as, $A_{free} \cong 0.000935A^2$. It may be noted that,

a) For A=45, Z=21 → A-2Z=3 and $A_{free} \cong 1.89 \cong 2$.
b) For A=63, Z=29 → A-2Z=5 and $A_{free} \cong 3.7 \cong 4$.
c) For A=112, Z=50 → A-2Z=12, $A_{free} \cong 11.7 \cong 12$.
d) For A=206, Z=82 → A-2Z=40, $A_{free} \cong 39.7 \cong 40$.
e) For A=235 Z=92, → A-2Z=51, $A_{free} \cong 51.6 \cong 52$.

Thus, by assuming a simple relation of the following form, proton number of the assumed (stable) mass number can be estimated approximately. It needs further study with respect to even-odd corrections.

$$\left.\begin{array}{l} (A-2Z) \approx 0.000935A^2 \\ Z \cong \left(\dfrac{A-0.000935A^2}{2}\right) \pm (0,2) \end{array}\right\} \tag{11}$$

Proceeding further, considering relation (10), maximum binding energy associated with any mass number can be expressed as,

$$\frac{BE}{A} \cong \frac{1}{A}\left\{A - 0.000935A^2 - A^{1/3} - A^{-1/2}\right\} 10.1\text{ MeV} \tag{12}$$

Following relations (10) and (12), currently believed Unified atomic mass unit [2,22] can be estimated in a very simple approach.

$$M_u c^2 \cong \left\{ \left(\frac{m_n c^2 + m_p c^2}{2} \right) - \text{mean} \left\langle \frac{BE}{A} \right\rangle_{A=4}^{A\approx 340} \right\} + m_e c^2 \tag{13}$$
$$\cong 938.918755 - \langle 7.956 \rangle + 0.511 \cong 931.473755 \text{ MeV}$$

where (m_n, m_p, m_e) represent neutron, proton and electron rest masses respectively. Now the most complicated Avogadro number [22,23] can be expressed as,

$$N_A \cong \left(\frac{1}{M_u c^2} \right) \frac{\text{atoms}}{\text{kg-mole}} \text{ Or } \frac{\text{atoms}}{\text{gram-mole}} \tag{14}$$
$$\cong 6.0222726 \times 10^{26} \frac{\text{atoms}}{\text{kg-mole}} \cong 6.0222726 \times 10^{23} \frac{\text{atoms}}{\text{gram-mole}}$$

With further study and estimating the average of maximum binding energy per nucleon of A=4 to 400, Avogadro number can be estimated accurately.

8. Discussion

Following our approach,

1) Even though they are having wide scope and very accurate, currently believed semi empirical mass formulae are having many complicated energy coefficients with different terms and different concepts [17-21]. We would like to emphasize the point that, clarity is missing in coupling or interpreting the terms and coefficients with strong and weak interactions. Similarly, energy coefficients associated with recently developed relativistic continuum Hartree-Bogoliubov (RCHB) theory having relativistic density functions are much more complicated [19].
2) Conceptually, relations (9) and (10) are very simple in understanding and having deep inner meaning.

   Relation (9) can be expressed as,

$$(BE)_{(Z,A)} \cong (A - A_{free} - A_{rad} - A_{asy}) 10.1 \text{ MeV} \tag{15}$$

Relation (10) can be expressed as,

$$(BE)_A \cong \left(A - A_{free} - A_{rad} - A_x\right) 10.1 \text{ MeV} \tag{16}$$

where A_x is a term that needs a review.

It needs a review with respect to A=4 and A > 200. We are working in this new direction. With even-odd corrections, shell corrections and other microscopic corrections, it can be refined.

3) Best possible range of super heavy atomic nuclides [24] can be understood and corresponding experimental design, set up and trial runs can be carried out with confidence.
4) Avogadro like most complicated numbers can be estimated in a very simplified approach.

9. Conclusion

We would like to emphasize the point that, strong and weak interactions play a vital role in basic nuclear structure and further study may help in exploring the atomic nucleus in a unified approach. Based on the above concepts and data presented in Table.1, Table. 2, Table. 3 and Fig. 1, it seems possible to understand super heavy mass numbers and maximum binding energy associated with any mass number with our 4G model of final unification. With further study, Unified atomic mass unit and Avogadro number can be estimated with best possible accuracy.

References

[1] Seshavatharam U.V.S and Lakshminarayana S.A very brief review on strong and electroweak mass formula pertaining to 4G model of final unification. Proceedings of the DAE Symp. on Nucl. Phys. 67, (2023), 1173.

[2] Seshavatharam U.V.S and Lakshminarayana S. Exploring condensed matter physics with refined electroweak term of the strong and electroweak mass formula. World Scientific News .193(2), (2024), 105-13.

[3] Seshavatharam U.V.S and Lakshminarayana S. Inferring and confirming the rest mass of electron neutrino with neutron lifetime and strong coupling constant via 4G model of final unification. World Scientific News 191, (2024), 127-156.

[4] Seshavatharam U. V. S., Gunavardhana Naidu T and Lakshminarayana S. To confirm the existence of heavy weak fermion of rest energy 585 GeV. AIP Conf. Proc. 2451, (2022), 020003.

[5] Seshavatharam U.V.S and Lakshminarayana S. 4G model of final unification – A brief report Journal of Physics: Conference Series 2197, (2022), 012029.

[6] Seshavatharam U.V.S and Lakshminarayana. Understanding nuclear stability range with 4G model of nuclear charge. World Scientific News. 177, (2023), 118-136.

[7] Seshavatharam U.V.S, S. Lakshminarayana, H. K. Cherop and K. M. Khanna, Three Unified Nuclear Binding Energy Formulae. World Scientific News, 163, (2022), 30-77.

[8] Seshavatharam U.V.S, Lakshminarayana, S., On the Combined Role of Strong and Electroweak Interactions in Understanding Nuclear Binding Energy Scheme. Mapana Journal of Sciences, 20(1), (2021),1-18.

[9] Seshavatharam U.V.S and Lakshminarayana S., Strong and Weak Interactions in Ghahramany's Integrated Nuclear Binding Energy Formula. World Scientific News, 161, (2021), 111-129.

[10] Seshavatharam U.V.S and Lakshminarayana S. Is reduced Planck's constant - an outcome of electroweak gravity? Mapana Journal of Sciences. 19(1), (2020), 1.

[11] Workman R.L. et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01, 2022.

[12] Ghiorso Albert. Einsteinium and Fermium. Chemical and Engineering News. 81 (36), (2003), 174–175.

[13] Kondev, F. G.; Wang, M.; Huang, W. J.; Naimi, S.; Audi, G. The NUBASE2020 evaluation of nuclear properties (PDF). Chinese Physics C. 45 (3), (2021), 030001.
[14] Peter Möller. The limits of the nuclear chart set by fission and alpha decay. EPJ Web Conf. 131, (2016), 03002.
[15] Oganessian, Yu & Rykaczewski, Krzysztof. A beachhead on the island of stability. Physics Today. 68, (2015), 32-38.
[16] Xin, YQ., Ma, NN., Deng, JG. et al. Properties of 114 super-heavy nuclei. NUCL SCI TECH 32, (2021), 55.
[17] Myers W. D. and Swiatecki W. J. Nuclear Properties According to the Thomas-Fermi Model.LBL-36557 Rev. UC-413, (1995)
[18] Myers W. D. and Swiatecki W. J. Table of nuclear masses according to the 1994 Thomas-Fermi model. United States: N. p., (1994). Web.
[19] X.W. Xia, Y. Lim, P.W. Zhao et al. The limits of the nuclear landscape explored by the relativistic continuum Hartree–Bogoliubov theory. Atomic Data and Nuclear Data Tables. 121–122, (2018), 1-215.
[20] Bethe H. A. Thomas-Fermi Theory of Nuclei. Phys. Rev., 167(4), (1968), 879-907.
[21] Gao, Z.P., Wang, YJ., Lü, HL. et al., Machine learning the nuclear mass. NUCL. SCI. TECH. 32, (2021), 109.
[22] Tiesinga, E., Mohr, P. J., Newell, D. B. & Taylor, B. N. CODATA recommended values of the fundamental physical constants: 2018. Rev. Mod. Phys. 93, (2021), 025010.
[23] Bengt Nordén. The Mole, Avogadro's Number and Albert Einstein. Molecular Frontiers Journal. 5, (2021), 66-78.
[24] J.M. Gates et al. Towards the Discovery of New Elements: Production of Livermorium (Z=116) with 50Ti. arXiv:2407.16079v1 [nucl-ex], (2024).

HADRONIC JOURNAL 48, 63 - 85 (2025) - 63 -
DOI 10.29083/HJ.48.01.2025/SC1

A NEW APPROACH TO CONCEIVE THE MEASUREMENT OF THE ONE-WAY SPEED OF LIGHT BASED ON AN ASTONISHING CONFLICT WITHIN RELATIVITY

Rodrigo de Abreu
Departamento de Física, Instituto Superior Técnico Universidade de Lisboa, 1049-001 Lisboa, Portugal
rabreu@tecnico.ulisboa.pt

Abstract

The measurement of the speed of light one-way can be easily conceived since we can consider a rod with length l_1 between the extremities A' and B' and emit light from A' to B' and reciprocally emit light from B' to A'. If we measure the time $A'B'$ and the time $B'A'$ we know the speed of light $A'B'$ and $B'A'$. However, standard approach affirm astonishingly that this is not possible. Because we need to know the speed of light to synchronize the clocks at A' and B', to measure the times $A'B'$ and $B'A'$. Why? Because standard approach accepts the necessity to have synchronized clocks. And astonishingly also affirm that the one-way speed of light one-way is the two-way speed of light measured in one-clock with the value c in vacuum. In the following approach we defend that this standard approach cannot subsist based on the conceptualization of the measurement one-way. For this we use a new method using a gap of synchronization that standard approach cannot be aware.

Keywords: simultaneity and synchronization, Abreu´s axiom, gap of synchronization, one-way speed of light, two-way speed of light, preferred frame, experimental determination, Relativity Principle, Einstein frame, Einstein synchronization, Lorentzian time, intrinsic desynchronization, Lorentz transformation, IST transformation, time dilation and contraction, Lorentz-FitzGerald contraction and dilation, Einstein simultaneity, new method, Einstein method, synchronized time, conventionalism controversy.

1. Introduction

"De Abreu proposed to abandon the Relativity Principle in favour of 'restricted Relativity Principle' that allows the absolute space with a preferred reference system, referred to as 'the Einstein's lost frame'. This idea was future developed in [De Abreu 2002, 2004; De Abreu & Guerra 2005; Guerra & de Abreu 2006]. The velocity relative to the preferred reference system is said to be the absolute velocity, and a velocity relative to non-preferred system is said to be the Einstein velocity [De Abreu 2004]. The starting point of De Abreu (and jointly with Guerra) is the observation that the Einstein synchronization of clocks can be made in one and only one reference system. Analysis of the clock synchronization (related to one-way versus two-ways light velocity) leads Authors to consider the abandoning of the Relativity Principle (that all reference systems are equivalent)." [1-13]
(https://ui.adsabs.harvard.edu/abs/2011arXiv1104.0682O/abstract)
Based on this new approach we develop a protocol to experimentally measure the one-way speed of light.

2. The Method

2.1 Simultaneity and Synchronization

If the simultaneity of the emissions of light at A' and B' can be conceived and operationally implemented, we can emit light at A' and B' simultaneously with the initialization of the clocks marking zero at A' and B'. Therefore, if this is so we can measure the light times $A'B'$ and $B'A'$ and therefore measure the one-way speed $A'B'$ and $B'A'$. Indeed, the clock A' measured the time of light emitted by B' and the clock B' measure the time of light emitted by A'. We have the problem solved. For achieve that we can use a generalization of Einstein method based on the time gap of the new approach [1-26].

2.2 Einstein Frame

Einstein affirm that the one-way speed of light is *c* in every frame. This cannot be so. Indeed, if we assume the existence of a frame where the one-way speed of light is isotropic with the value *c*, the value measured for the two-way speed of light, the frame designated by us [5] Einstein Frame (*EF*), for this frame we can synchronize clocks at distance with light using Einstein method. Therefore, if we assume the existence of *EF* than for a frame moving with velocity v_1 in relation to that frame we calculate the speed of light one-way and this speed cannot be *c* [1, 6]. Zbigniew Oziewicz also emphasize "Abreu's Axiom", the independence of the speed of light in *EF* of the speed of the frame of the emitter, the source of light [1].

Since the speed of light one-way cannot be *c* in every frame the axiom of the constancy of the one-way speed of light cannot subsist and we must construct another theory. This has been accomplished [1-26]. Based on the existence of Einstein Frame. The unique frame where the speed of light is isotropic with the value *c*.

2.3 The Synchronization of Clocks for a frame moving with speed v_1 in relation to Einstein Frame.

If we consider a rod *A′B′* moving with speed v_1 in relation to *EF* and if we calculate the speed of light *A′B′* and *B′A′* we obtain [6],

$$c'_{A'B'}=\frac{c-v_1}{1-\frac{v_1^2}{c^2}}=\frac{c}{1+\frac{v_1}{c}} \qquad (1)$$

and

$$c'_{B'A'}=\frac{c+v_1}{1-\frac{v_1^2}{c^2}}=\frac{c}{1-\frac{v_1}{c}} \qquad (2)$$

This is the speed of light based on the Lorentz-FitzGerald contraction and Time dilation valid only in relation to *EF* [3, 4]. Therefore, we can calculate the times *A´B´* and *B´A´* since we can synchronize the clocks with light using the real speed that is not *c*. We know that the one-way speed of light is not *c* (isotropic) except in *EF*. This is not only an experimental problem. *The experimental verification cannot be yet achieved*. And an experiment only can corroborate or not a theory [1-121]. However, we know that standard approach contradicts itself, it is not internally consistent. Since if we assume the existence of a frame where the one-way speed of light is *c* for another frame the one-way speed of light cannot be *c*.

Indeed, when light is emitted from *A´* to *B´* and the clock at *B´* is marking $\frac{l_1}{c}\left(1+\frac{v_1}{c}\right)$ when light arrive at *B´* the clock is synchronized with the clock at *A´* (from eq. (1)). Therefore, we have the clocks at *A´* and *B´* marking the same time. And after *10 s* the clocks continue synchronized and can be resected to zero (and continue synchronized) and emit light to the other. And we obtain for the times *A´B´* and *B´A´*

$$T'_{A'B'}=\frac{l_1}{c}\left(1+\frac{v_1}{c}\right)=\frac{l_1}{c}+\frac{l_1}{c}\frac{v_1}{c}=\frac{l_1}{c}+\delta \qquad (3)$$

$$T'_{B'A'}=\frac{l_1}{c}\left(1-\frac{v_1}{c}\right)=\frac{l_1}{c}-\frac{l_1}{c}\frac{v_1}{c}=\frac{l_1}{c}-\delta \qquad (4)$$

Then we have for the two-way speed of light the value *c*,

$$T=T'_{A'B'}+T'_{B'A'}=\frac{2l_1}{c} \qquad (5)$$

As expected, and consistently we obtain for a first order approximation the classic result (see eq. (1) and (2)) contrary to the

standard relativistic approach that is rigorously *c* – does not depend on v_1. With this new approach based on the existence of *EF* we show that the standard approach affirming the constancy of the one-way speed of light cannot subsist [3, 4, 6]. And with this new approach we can develop a method to synchronize the clocks also in the frame with velocity generic v_1. Although we don´t know v_1 *ab initio*. This is possible because we previously discover the existence of a conceptualization of simultaneity and synchronization different of the obtained in *EF*, that the standard formulation cannot be aware as Zbigniew Oziewicz refer (Einstein synchronization is valid only in *EF*). Indeed when v_1 is different from zero we have a gap of "synchronizations" and "simultaneities" that permit the extension of the conceptualization and consequent experimental implementation of this gap.

We can introduce the several values of the one-way speed of light between zero and a generic v_1 by

$$c'_{A'B'}=\frac{c}{1+\frac{\propto v_1}{c}} \qquad (6)$$

and

$$c'_{B'A'}=\frac{c}{1-\frac{\propto v_1}{c}} \qquad (7)$$

with $\propto \epsilon\ [0,1]$. Therefore, we have a gap of "simultaneities" and synchronizations" since we have emissions and receptions between zero and 1.

Therefore, we have

$$T'_{A'B'}=\frac{l_1}{c}\left(1+\frac{\propto v_1}{c}\right)=\frac{l_1}{c}+\frac{l_1}{c}\frac{\propto v_1}{c}=\frac{l_1}{c}+\delta \qquad (8)$$

and

$$T'_{B'A'}=\frac{l_1}{c}\left(1-\frac{\propto v_1}{c}\right)=\frac{l_1}{c}-\frac{l_1}{c}\frac{\propto v_1}{c}=\frac{l_1}{c}-\delta \quad (9)$$

And we maintain the two-way speed of light for the several “synchronizations” begin with “Einstein synchronization” (with $\propto = 0$) the assumption that the one-way speed of light is really *c*.

$$T=T'_{A'B'}+T'_{B'A'}=\frac{2l_1}{c} \quad (10)$$

Therefore, it is very simple. At *B´* the clock is waiting the arrival of light from *A´* marking “Einstein synchronization” $\frac{l_1}{c}$. After *10 s* the clocks are resected to zero and if they are really synchronized the times measured are

$$T'_{A'B'}=\frac{l_1}{c}\left(1+\frac{0\times v_1}{c}\right)=\frac{l_1}{c} \quad (11)$$

$$T'_{B'A'}=\frac{l_1}{c}\left(1-\frac{0\times v_1}{c}\right)=\frac{l_1}{c} \quad (12)$$

Of course, the speed of light is the same, but this “synchronization” is a desynchronization (when $v_1 \neq 0$) that originates the experimental result $\frac{l_1}{c}$.

It is obvious that we can introduce a drift correspondent to other values of $\propto$ and only when $\propto = 1$ we obtain the correct result. And for this real synchronization we obtain the last two-way speed of light with the protocol. If this is so this can be observable and the value of v_1 can be experimentally conceived. Indeed we can

understand that if we initiate the experimental process with a rod with length l_1 between A' and B' and if we consider that rod moving with v_1 in relation to EF and if we emit light at A' initiating the clock at A' marking zero and at B' we can have a Lorentzian clock waiting marking $\frac{l_1}{c}$ that is initiated by the arrival of light emitted by A'. If the clock at B' is really synchronized must mark $\frac{l_1}{c}(1+\frac{v_1}{c})$. Therefore, we can consider the two clocks that are initiated by the arrival of light from A'. Therefore, when light arrive both clocks initiate and of course mark two different times. Since the speed of light is $c/(1+\frac{v_1}{c})$ the clock that is synchronized is the synchronized clock marking $\frac{l_1}{c}(1+\frac{v_1}{c})$ and not the other as Einstein stated, a Lorentzian clock. When we consider the reset to zero at A' after 10s at B' there is not reset yet with the Lorentzian clock and therefore at B' we don´t have emission yet. Therefore, the emission at B' correspond to other position of the rod in EF. The emissions are not really simultaneous although similar since we proceed with Lorentzian clocks as with synchronized clocks. This is Einstein simultaneity since Einstein affirm that Lorentzian clocks are synchronized. But it is operational, we don´t need to know v_1. The emissions are not simultaneous and therefore we obtain $\frac{l_1}{c}$ at the receptions as we expect if the speed of light was *c*. It seems, but it is not. The introduced desynchronization originates the experimental result observed but does not signify the *c* value assumed by Einstein. The reset to zero is local and because of that when we reset to zero l_1 is not in the same position correspondent to the position of the rod when the clock at A' reset, the emissions are not simultaneous and therefore we have the difference of times for the several "synchronizations"

$$T_{A'B'}=\frac{l_1}{c}\left(1+\frac{\propto v_1}{c}\right) \qquad (13)$$

$$T'_{B'A'}=\frac{l_1}{c}\left(1-\frac{\propto v_1}{c}\right) \qquad (14)$$

$$T = T'_{A'B'} + T'_{B'A'} = \frac{2l_1}{c} \qquad (15)$$

What we need to know and don´t know yet is for $\propto = 1$. But when experimentally the gap is exceeded, we don´t obtain

$$T = T'_{A'B'} + T'_{B'A'} = \frac{2l_1}{c} \qquad (16)$$

Therefore, it is through experiment that we obtain v_1. It is "ontic". It is not conventional. Of course, this is so in a new context of Relativity. With 3 frames. And one is the unique Einstein Frame.

Conclusion

We consider the emissions from the extremities A' and B' of a rod moving with velocity v_1 in relation to Einstein Frame (*EF*), the frame with isotropy of the one-way speed of light *c*. Since the formulation must consider *EF* with $v_1 = 0$ we need to consider 3 frames, and this is a new result that evince why the standard formulation is internally inconsistent. It is crucial to understand the difficulties that originates on the terminological confusions of relativity and construct a new language. We obtain new results when we consider the mathematical relations between two moving frames with two velocities v_1 and v_2 in relation to *EF* with $v = 0$. We obtain a gap of desynchronizations to the times at A' and B' with the extremes at A' and B' with synchronized clocks. We can define this gap with a parameter $\propto$ between *zero and one* that we obtain from the several values of the velocities between $zero\ and\ v_1$. We can therefore experimentally discover the values of v_1 when experimentally the gap is surpassed. Since the observed value of the tway speed of light does not satisfy the internal value of the gap. This solves also the controversy about conventionalism.

References

1. Zbigniew Oziewicz, How do you add relative velocities? http://www.naturalphilosophy.org/pdf/group25.pdf

2. Zbigniew Oziewicz, Ternary relative velocity; astonishing conflict of the Lorentz group with relativity, Vladimir Gladyshev, Editor, Physical Interpretations of Relativity Theory, Moscow 2007, pages 292-303, ISBN 978-5-7038-3178-6 https://arxiv.org/abs/1104.0682 https://ui.adsabs.harvard.edu/abs/2011arXiv1104.0682O/abstract

3. de Abreu, R. (2024) THE ASTONISHING CONFLICT OF TIME DILATION WITHIN RELATIVITY, https://scholar.tecnico.ulisboa.pt/records/c0sbEhuRpsLz7fzP0v7p1pETkipmIXkL1ImS?lang=pt https://hadronicpress.com/HJ/HJVol/HJ47-3I.php

4. Mateljević M (2024) Lorentz Transformation and time dilatation. Ann Math Phys 7(1): 016-022. DOI: 10.17352/amp.000104

5. de Abreu, R. The Relativity Principle and the Indetermination of Special Relativity Ciência & Tecnologia dos Materiais, vol. 14, nº 1, p. 74 (2004), https://vixra.org/abs/2301.0139

6. de Abreu, R. (2024) The Astonishing Conflict of the Constancy of the One-Way Speed of Light within Relativity https://vixra.org/pdf/2402.0086v1.pdf

7. de Abreu, R. and Guerra, V. Eur. J. Phys. 30, 2 (2009) Special relativity as a simple geometry problem - IOPscience http://web.ist.utl.pt/d3264/papers/AG2009.pdf https://ui.adsabs.harvard.edu/abs/2009EJPh...30..229D/abstract

8. de Abreu, R. The physical meaning of synchronization and simultaneity in Special Relativity
https://arxiv.org/abs/physics/0212020
http://ui.adsabs.harvard.edu/abs/2002physics..12020D/abstract

9. de Abreu, R. Reinterpretation of Lorentz Transformation and Resolution of Relativity Paradoxes
http://arxiv.org/abs/physics/0203025 ; EPS-12 Trends in Physics, Abstracts p. 270, Budapest (2002).
http://vixra.org/abs/1505.0065
https://www.semanticscholar.org/paper/Reinterpretation-of-Lorentz-Transformation-and-of-Abreu/0f708d7f47b9fbd8bfe49e81bbbc1fcb33cecbb9
https://ui.adsabs.harvard.edu/abs/2002physics...3025D/abstract

10. de Abreu, R. Deduction of Lorentz Transformation from the existence of absolute rest. Deduction of the speed of light in any frame of reference https://arxiv.org/abs/physics/0210023
https://ui.adsabs.harvard.edu/abs/2002physics..10023D/abstract

11. Guerra, V. and de Abreu Foundations of Physics, On the Consistency between the Assumption of a Special System of Reference and Special Relativity Vol. 36, No. 12, December (2006) On the Consistency between the Assumption of a Special System of Reference and Special Relativity | SpringerLink
https://ui.adsabs.harvard.edu/abs/2006FoPh...36.1826G/abstract

12. de Abreu, R. and Guerra, V. Eur. J. Phys. 29, 1 (2007) The principle of relativity and the indeterminacy of special relativity - IOPscience
https://iopscience.iop.org/article/10.1088/0143-0807/29/1/004
The principle of relativity and the indeterminacy of special relativity (ulisboa.pt)
https://ui.adsabs.harvard.edu/abs/2008EJPh...29...33D/abstract

13. Burde, G. I. Special Relativity with a Preferred Frame and the Relativity Principle
Journal of Modern Physics vol. 9, Nº 8 (2018)
https://doi.org/10.4236/jmp.2018.98100
https://ui.adsabs.harvard.edu/abs/2016arXiv161008771B/abstract

14. de Abreu, R. Gazeta de Física, vol. 21, Fasc. 3 (1998)
https://www.spf.pt/magazines/GFIS/398/1153

15. Homem, G. Physics in a synchronized space-time. Master's thesis, Instituto Superior Técnico, Universidade Técnica de Lisboa, (2003)

16. Homem, G. On Abreu's theory of space and time and new measurements of absolute velocities
https://ui.adsabs.harvard.edu/abs/2002physics..12050H/abstract
https://www.semanticscholar.org/paper/On-Abreu%27s-theory-of-space-and-time-and-new-of-Homem/d058637f19a2b25816cf47d034a8b23b898acee3

17. de Abreu, R and Guerra, V. (2005) [physics/0512196] Is the assumption of a special system of reference consistent with Special Relativity? (arxiv.org)

18. de Abreu, R. and Guerra, V. Relativity – Einstein's Lost Frame (Extra]muros[, Lisboa, 2005), 1st ed.
http://web.tecnico.ulisboa.pt/vguerra/papers/relshort.pdf

19. Guerra, V. and de Abreu, R. (2006) Special Relativity in Absolute Space: from a contradiction in terms to an obviousness https://arxiv.org/abs/physics/0603258
https://ui.adsabs.harvard.edu/abs/2006physics...3258G/abstract

20. Guerra, V. and de Abreu, R. Eur. J. Phys. 26, 6 (2005)
The conceptualization of time and the constancy of the speed of light – IOPscience
https://arxiv.org/abs/physics/0504107

https://web.ist.utl.pt/ist13264/papers/ejp5_6_s05.pdf
https://ui.adsabs.harvard.edu/abs/2005EJPh...26S.117G/abstract

21. Guerra, V. and de Abreu, R. Time, Clocks and the Speed of Light
https://ui.adsabs.harvard.edu/abs/2006AIPC..861.1103G/abstract

22. de Abreu, R. and Guerra, V. EJTP Vol. 12 Issue 34, p183 (2015)
http://www.ejtp.com/articles/ejtpv12i34p183.pdf
Electronic Journal of Theoretical Physics (ejtp.com)

23. de Abreu, R. Guerra, V. Speakable and Unspeakable in Special Relativity: The Ageing of the Twins in the Paradox
http://vixra.org/abs/1804.0320
https://web.ist.utl.pt/vguerra/papers/Ref2.pdf

24. de Abreu, R. Guerra, V. The Resolution of the Twin Paradox in a One-Way Trip http://vixra.org/abs/1805.0126

25. de Abreu, R. On the Experimental Determination of the One-Way Speed of Light
https://scholar.tecnico.ulisboa.pt/records/l8ZmPhf_htHQS13q9G3q7B4h5dYFJdzLU2Ri?lang=en
https://vixra.org/abs/1808.0646
https://www.semanticscholar.org/paper/On-the-Experimental-Determination-of-the-One-Way-of-Abreu/f8df86a10541bf42fa3b024d548fe55ebcb11bfb?citedSort=relevance&citedPage=2

26. Einstein, A. Ann. Phys. 17, 132 (1905): "On the Electrodynamics of Moving Bodies", "Einstein´s Miraculous Year, Five Papers That Changed the Face of Physics" Edited and Introduced by John Stachel, Princeton University Press (1998)

27. Consoli, M. Pluchino, A. Atti della Accademia Peloritana dei Pericolanti Vol. 96, No. S1, A2 (2018)

28. Kittel, C. Am. J. Phys. 42, 726 (1974) Larmor and the Prehistory of the Lorentz Transformations https://doi.org/10.1119/1.1987825

29. Andersen, F. PhD Thesis, NMBU (2017) https://nmbu.brage.unit.no/nmbu-xmlui/handle/11250/2500054

30. Myrstad, J. A. Borderology: Cross-disciplinary Insights from the Border Zone, p. 93, Springer (2019) DOI: 10.1007/978-3-319-99392-8_8 http://dx.doi.org/10.1007/978-3-319-99392-8_8

31. Consoli, M. Pluchino, Michelson-Morley Experiments: An Enigma For Physics And The History Of Science, World Scientific (2019) https://www.worldscientific.com/worldscibooks/10.1142/11209

32. Consoli, M. Pluchino, A. The CMB, Preferred Reference System, and Dragging of Light in the Earth Frame, Universe (2021) https://doi.org/10.3390/universe7080311

33. Haug, E The Return of Absolute Simultaneity? A New Way of Synchronizing Clocks Across Reference Frames http://vixra.org/abs/1605.0057

34. Haug, E A Motion Paradox from Einstein's Relativity of Simultaneit http://vixra.org/abs/1711.0408

35. de Abreu, R. Comment on "A Motion Paradox from Einstein´s Relativity of Simultaneity" http://vixra.org/abs/1810.0452

36. Geloni, G. Kocharyan, V. Saldin, E. https://arxiv.org/abs/1601.07738

37. Geloni, G. Kocharyan, V. Saldin, E. https://arxiv.org/abs/1610.04139

38. Geloni, G. Kocharyan, V. Saldin, E. https://arxiv.org/abs/1704.01843

39. de Abreu, R. Guerra http://vixra.org/abs/1906.0312 Comment on “Misconceptions Regarding Conventional Coupling of Fields and Particles in XFEL Codes” https://www.semanticscholar.org/paper/Comment-on-%E2%80%9CMisconceptions-Regarding-Conventional-Abreu-Guerra/9f47eda7665693ba5717f603eb46ee46fd6e7ae1

40. Mansouri, R. and Sexl, R. Gen. Relat. Gravit. 8, 497 (1977)

41. Gianfranco Spavieri *et al* 2018 *J. Phys. Commun.* **2** 085009

42. Spavieri, G. PAIJ vol.1, Issue 1 (2017)

43. Consoli, M. Pluchino, A. Eur. Phys. J. Plus 133:295 (2018)

44. Ricou, M. On the speed of light Physics Essays, Vol. 30, 461-468 (2017)

45. de Abreu, R. Guerra, V. Comment on “On the speed of light” by Manuel Ricou http://vixra.org/abs/1906.0310

46. Wutke, A. https://www.researchgate.net/publication/326672264_Absolute_Simultaneity_and_Rest_Consistent_with_Special_Relativity_Science_or_Philosophy

47. Chandru Iyer Comment on 'Thought experiment discriminating special relativity from preferred frame theories' 2018 *J. Phys. Commun.* **2** 118001

48. Potvin, G. Ether Interpretation of General Relativity, RESEARCHERS.ONE (2018) https://www.researchers.one/article/2018-12-7

49. Grøn, Ø. Eur. J. Phys. 27, 885 (2006) https ://iopscience.iop.org/article/10.1088/0143-0807/27/4/019

50. Iyer, C. Harmonic mean, the Gamma factor and Speed of Light https://arxiv.org/abs/0811.0785

51. Iyer, C. Comment on "The Principle of Relativity and the Indeterminacy of Special Relativity" Eur. J. Phys. 29, 4 (2008) https://iopscience.iop.org/article/10.1088/0143-0807/29/4/L01/meta

52. Iyer, C. Prabhu, G. A constructive formulation of the one-way speed of light, Am. J. Phys. 78 (2) (2010) https://www.researchgate.net/publication/45895243_A_constructive_formulation_of_the_one-way_speed_of_light

53. Iyer, C. Prabhu, G. Time Dilation and the Equivalence of Inertial Frames https://arxiv.org/abs/0710.1594

54. Iyer, C. Prahbu, G. The One-Way Speed of Light: A Simple Formulation https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=17f30a26d1ba7006c9a1afb392990a762b18e5af

55. Moller, C. Proc. of The Royal Soc. A vol. 270, Issue 1342 (1960) https://doi.org/10.1098/rspa.1962.0220

56. Schwartz, H. M. Am. J. Phys. 39, 1269 (1971) https://doi.org/10.1119/1.1976621 A New Method of Clock Synchronization without Light Signals

57. Sears, F. W. Am. J. Phys. 37, 668 (1969) https://doi.org/10.1119/1.1975747 Simultaneity without synchronized clocks

58. Ramakrishnan, A. Journal of Math. Anal. and App. 249, 243 (2000)

59. Bricmont, J. https://arxiv.org/pdf/1703.00294.pdf History of Quantum Mechanics or the Comedy of Errors

60. Giovanelli, M. https://www.researchgate.net/publication/338680431_Lorentz_Contraction_vs_Einstein_Contraction_Reichenbach_and_the_Philosophical_Reception_of_Miller'sEther-Drift_Experiments

61. Mamone-Capria, M. Journal for Foundations and Applications of Physics, vol.5, Nº 2, 163 (2018) [1704.02587] On the Incompatibility of Special Relativity and Quantum Mechanics (arxiv.org)

62. Spavieri, G. Gillies, G. T. Haug, E. G., Sanchez, A. (2019): Light propagation and local speed in the linear Sagnac effect, Journal of Modern Optics, http://doi: 10.1080/09500340.2019.1695005

63. Unnikrishnan, C. The Theories of Relativity and Bergson's Philosophy of Duration and Simultaneity During and After Einstein's 1922 Visit to Paris. *Preprints* **2020**,2020010273 http://doi: 10.20944/preprints202001.0273.v1

64. Spavieri, G. Guerra, V. de Abreu, R. Gillies, G. T Eur. Phys. J. D. 47, 457 (2008)

65. Spavieri, G. Gillies, G. Haug, E. G., Sanchez, A. Applied Physics Research Vol. 11, No. 4 (2019)

66. Spavieri, G. and Haug, E. G. Why the Sagnac effect favors absolute over relative simultaneity Physics Essays**,** Vol. 32, 331-337 (2019)

67. Spavieri, G. Gillies, G. T. Haug, E. G. Journal of Modern Optics, vol. 68, Issue 4 (2021)

68. Salmon, W., 1977. "The Philosophical Significance of the One-Way Speed of Light," *Noûs*, 11: 253–292

69. https://plato.stanford.edu/entries/spacetime-convensimul/

70. Perez, I. https://arxiv.org/abs/1102.4837 On the experimental determination of the one-way speed of light

71. Guerra, V. de Abreu, R. Comment on: "From classical ether-drift experiments: the narrow window for a preferred frame" [Phys. Lett. A 333 (2004) 355] Phys. Lett. A 361, Issue 6 (2007)

72. Lewis, G. F. Barnes, L. A. The One-Way Speed of Light and the Milne Universe Publication of the Astronomical Society of Australia, vol. 38, (2021) https://arxiv.org/abs/2012.12037

73. R.J. Buenker, GPS-Compatible Lorentz Transformation that Satisfies the Relativity Principle, Journal of Astrophysics and Aerospace Technology, 3: 115. DOI: 10.4172/2329-6542.1000115.

74. Netchitailo, V. S. https://vixra.org/abs/2012.0222

75. Pagano, A., Pagano, E.V. EPJ H 44, 321-330 (2019 https://doi.org/10.1140/epjh/e2019-90058-4

76. Greaves, E. D., Michel Rodrigues, A., Ruiz-Camacho, J. Am. J. Phys. 77, 894 (2009) https://doi.org/10.1119/1.3160665

77. de Abreu, Guerra, V. Comment on “A one-way speed of light experiment” (2009) https://www.researchgate.net/publication/45886873

78. Leaf, B. Philosophy of Science, vol. 22, Number 1 (1955) https://doi.org/10.1086/287387

79. Spavieri, G. Quintero, J. Unnikrishnan, C. Gillies, G. Phys. Lett. A 376(s 6-7):795-797 (2012) DOI: 10.1016/j.physleta.2012.01.010

80. Scott Blair, G. Relativity and Indeterminacy. *Nature* **170,** 582 (1952). https://doi.org/10.1038/170582a0

81. Hines, C. Relativity and Indeterminacy. *Nature* **170,** 582 (1952). https://doi.org/10.1038/170582a0

82. Mayantz, L. The enigma of probability in physics https://inis.iaea.org/search/search.aspx?orig_q=RN:18074404

83. Mayantz, L Beyond the quantum paradox, Taylor & Francis (1994)

84. Medvedev, S. Yu. Progress in Physics, vol. 10, 151 (2014) http://www.ptep-online.com/2014/PP-38-04.PDF

85. Del Santo, F. Gisin, N. http://philsci-archive.pitt.edu/id/eprint/18601 (2021)

86. Fleming, A. Matveev, V. Matvejev , O. https://www.researchgate.net/publication/264851293_Self-field_Theory_and_General_Physical_Uncertainty_Relations

87. Hill, J. Cox, B. Proc. Royal Soc. A (2012) https://doi.org/10.1098/rspa.2012.0340

88. Leubner, C. Aufinger, K. Krumm, P. Eur. J. Phys. 13, 170 (1992)

89. Drągowski, M., Włodarczyk, M. The Doppler Effect and the Anisotropy of the Speed of Light. *Found Phys* **50,** 429–440 (2020). https://doi.org/10.1007/s10701-020-00337-5

90. Ronald Anderson, S.J., Stedman, G.E. Distance, and the conventionality of simultaneity in special relativity. *Found Phys Lett* **5,** 199–220 (1992). https://doi.org/10.1007/BF00692800

91. de Abreu, R. Simultaneity and Synchronization by Rods as a Simple Geometry Problem https://vixra.org/abs/2103.0196

92. Dias Ferreira, L. Criticism to the Twin´s Paradox, Colégio Valsassina, Lisbon, Portugal Universal Journal of Physics and Application Vol. 15(1), pp. 1 - 7 DOI: 10.13189/ujpa.2021.150101 **Reprint (PDF) (412Kb)**

93. de Abreu, R. Comment on "Criticism to the Twin´s Paradox" by Luís Dias Ferreira https://vixra.org/abs/2107.0077

94. Borah, B. K. Schwarzschild-like solution for the gravitational field of an isolated particle on the basis of 7- dimensional metric, International Journal of Scientific and Research, vol. 3, Issue 10, October 2013

95. Borah, B. K. An Approach to New Concept of Time on the Basis of Four Fundamental Forces of Nature, International Journal of Scientific and Research Publications, vol. 3, Issue 6, June 2013

96. Borah, B. K. Unification of Four Fundamental Forces of Nature Developing 7- Dimensional Metric on The Basis of New Concept of Time https://www.worldwidejournals.com/international-journal-of-scientific-research- (IJSR)/article/unification-of-four-fundamental-forces-of-nature-developing-7andndash-dimensional-metric-on-the-basis-of-new-concept-of-time/NjU0OQ==/?is=1&b1=33&k=9

97. de Abreu, R. The Energy-Entropy Principle https://arxiv.org/abs/physics/0207022

https://ui.adsabs.harvard.edu/abs/2002physics...7022D/abstract

98. de Abreu, R. Guerra, V. The concepts of work and heat and the first and second laws of thermodynamics https://arxiv.org/abs/1203.2294

99. de Abreu, R. Guerra, V. Introducing thermodynamics through energy and entropy Am. J. Phys. 2012 https://aapt.scitation.org/doi/10.1119/1.3698160

https://ui.adsabs.harvard.edu/abs/2012AmJPh..80..627D/abstract

100. Caratheodory, C. https://greatestgreeks.wordpress.com/2016/12/12/constantine-caratheodory/

101. Caratheodory, C. https://www.researchgate.net/publication/312057731_On_Caratheodory's_approach_to_relativity_and_its_relation_to_hyperbolic_geometry

102. de Abreu, R. The Concept of Mass as a Quantity Derived from Motion. https://vixra.org/abs/1505.0094 https://www.semanticscholar.org/paper/The-Concept-of-Mass-as-a-Quantity-Derived-from-Abreu/6b203d93a115daa6649b6826963f0a31dd95c368

103. Mohazzabi, P. and Luo, Q. (2021) Has the Twin Paradox Really Been Resolved?. *Journal of Applied Mathematics and Physics*, **9**, 2187-2192. doi: 10.4236/jamp.2021.99138.

104. de Abreu, R. Comment on "Has the Twin Paradox Really Been Resolved?", viXra.org e-Print archive, viXra:2209.0045

105. de Abreu, R. Simultaneity and Synchronization, the Preferred Frame and the Principle of Relativity, viXra.org e-Print archive, viXra:2204.0060

106. Reference:https://www.physicsforums.com/threads/potential-energy-formula- in-special-relativty.991687/

107. Reichenberger, Andrea (2018) The Clock Paradox: Luise Lange´s Discussion REITCP-3 (philarchive.org)

108. Iyer, C. (2022) Importance of Synchronization in the observation of event coordinate DOI:10.36227/techrxiv.21657026

109. de Abreu, R. (2023) Proper Times, Time Dilation, Lorentz-FitzGerald Contraction and Distance, Time Ageing, Time Dilation-like and the Twin Paradox Conundrum Proper Times, Time Dilation, Lorentz-FitzGerald Contraction and Distance, Time Ageing, Time Dilation-like and the Twin Paradox Conundrum, viXra.org e-Print archive, viXra:2301.0029

110. de Abreu, R. (2023) An interesting case of Twins Paradox – Twins Approaching Each Other https://vixra.org/abs/2308.0109

111. de Abreu, R. (2023) The Time Dilation Conundrum and the Ageing of the Twins https://rxiv.org/abs/2301.0086

112. de Abreu, R. (2023) Simultaneity and Synchronization and Einstein´s Relative Simultaneity https://vixra.org/abs/2303.0053

113. Choi, Yang-Ho (2022) Multiple velocity composition in the standard synchronization https://ui.adsabs.harvard.edu/abs/2022OPhy...20...17C/abstract

114. Choi, Yang-Ho (2008) A New Approach to Special Relativity and The Universe https://arxiv.org/pdf/0801.2847

115. Choi, Yang-Ho (2019) https://link.springer.com/article/10.3938/jkps.75.176

116. Lambare, Justo On The Sagnac Effect and the Consistency of Relativity Theory https://doi.org/10.32388/UL1ZWZ

117. de Abreu, R. (2023) The Resolution of the Twin Paradox with Three Frames of Reference - A Mathematical and Physical Report https://vixra.org/abs/2307.0052

118. de Abreu, R (2002) The First Principle of Thermodynamics and the Non-Separability of the Quantities "Work" and "Heat": The adiabatic piston controversy https://ui.adsabs.harvard.edu/abs/2002cond.mat..5566D/abstract

119. R.M. Santilli, Lie-isotopic Lifting of Special Relativity for extended Deformable Particles, , Lettere Nuovo Cimento} { 37}, 545-555 (1983),\\ www.santilli-foundation.org/docs/Santilli-50.pdf

120. R. M. Santilli, "Compatibility of Arbitrary Speeds with Special Relativity Axioms for Interior Dynamical Problems," American Journal of Modern Physics, {\bf 5}, 143 (2016),\\http://www.santillifoundation.org/docs/ArbitrarySpeeds.pdf

121. H. Ahmar, G. Amato, J. V. Kadeisvili, J. Manuel, G. West, and O. Zogorodnia, "Additional experimental confirmations of Santilli's IsoRedShift and the consequential lack of expansion of the universe, "Journal of Computational Methods in Sciences and Engineering,{\bf 13}, 321 (2013),\\ http://www.santilli-foundation.org/docs/IRS-confirmations-212.pdf

HADRONIC JOURNAL 48, 87 - 123 (2025)
DOI 10.29083/HJ.48.01.2025/SC87

VARIATIONAL FORMULATIONS FOR A CHEMICAL REACTION AND RELATED MODELS IN SUPERCONDUCTIVITY INCLUDING AN INTERNAL ENERGY CONTEXT

Fabio Silva Botelho
Department of Mathematics
Federal University of Santa Catarina – UFSC
Florianópolis, SC, Brazil
fabio.botelho@usfc.br

Received October 3, 2024

Abstract

In its first part, this article develops a variational formulation for modeling a chemical reaction suitable to represent a combustion process. The results are obtained through standard tools of calculus of variations and optimization theory in function spaces. We assume such a chemical reaction develops in a control volume which allows the entering and leaving of the concerned reacting chemical substances. We highlight the related fluid motion is addressed in an Eulerian context.

Finally, in the last sections, we present variational formulations for models in superconductivity, including a magnetic field and respective magnetic potential. We highlight in such last sections it is included an internal variables approach as well.

AMS Classification 76A02, 76N15, 35Q56
Keywords. Variational formulations, chemical reaction, models in superconductivity.

1 Introduction

This article develops a variational formulation for a chemical reaction in a control volume in which is modeled a related Eulerian fluid motion in a non-relativistic context.

We consider the chemical reaction of a gaseous substance A with another gaseous substance B resulting in a third gaseous substance C.

More specifically, a unit fraction mass α_A of A reacts with a unit fraction mass α_B of B, resulting in a unit mass of a substance C, that is,

$$\alpha_A + \alpha_B = 1.$$

For modeling such a reaction, we consider a control volume denoted $\Omega \subset \mathbb{R}^3$, which corresponds to an open, bounded and connected set with a regular (Lipschitzian) boundary denoted by $\partial\Omega$.

The entire flow process develops along a time interval $[0, t_f]$, where $t \in [0, t_f]$ denotes time.

The results are obtained through standard tools of calculus of variations, basic constrained optimization and related Lagrange multipliers approach in infinite dimensional spaces.

As standard references in theoretical fluid mechanics we would cite [1, 2, 3, 4, 5, 6]. For related numerical methods we would mention [7, 8, 9]. Concerning similar models, we would cite [10, 11, 12, 13, 14].

Finally, details on the Sobolev spaces involved may be found in [15].

2 The main variational formulation

In this section, we develop in details the concerning variational formulation for a compressible case.

Assuming the Einstein convection of summing up repeated indices and denoting by

$$\mathbf{r}(x,t) = (X_1(x,t), X_2(x,t), X_3(x,t))$$

the macroscopic position field for such a compressible motion, we also assume the mass conservation equation as a constraint, which stands for

$$\frac{d\rho}{dt} + \rho \frac{\partial}{\partial x_j}\left(\frac{\partial X_j(x,t)}{\partial t}\right) = 0, \text{ in } \Omega \times [0, t_f].$$

Moreover, we suppose a non-constant field of temperature $T(\mathbf{r}(x,t), t)$ and a non-constant fluid density $\rho = \rho(\mathbf{r}(x,t), t) > 0$ so that the macroscopic system

kinetics energy functional stands for

$$E_c = \frac{1}{2}\int_0^{t_f}\int_\Omega \rho \mathbf{v}(\mathbf{r}(x,t),t)\cdot\mathbf{v}(\mathbf{r}(x,t),t)\,dx\,dt.$$

For such a modeling, denoting by $\mathbf{g} = -g\mathbf{k}$ the gravity field, we assume such a system is a particle one, comprised by a number of particles to be specified, with position fields

$$(\mathbf{r}_A)_j(x,t) = \mathbf{r}(x,t) + (\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t),\ \forall j \in \{1,\cdots,(N_A)_0(t)\},$$

for the substance A,

$$(\mathbf{r}_B)_k(x,t) = \mathbf{r}(x,t) + (\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t),\ \forall k \in \{1,\cdots,(N_B)_0(t)\},$$

for the substance B, and

$$(\mathbf{r}_C)_l(x,t) = \mathbf{r}(x,t) + (\hat{\mathbf{r}}_C)_l(\mathbf{r}(x,t),t),\ \forall l \in \{1,\cdots,(N_C)_0(t)\},$$

for the substance C.

The corresponding number of particles are

$$N_A(t) = \sum_{j=1}^{(N_A)_0(t)} (N_A)_j(t),$$

$$N_B(t) = \sum_{j=1}^{(N_B)_0(t)} (N_B)_j(t),$$

and

$$N_C(t) = \sum_{j=1}^{(N_C)_0(t)} (N_C)_j(t).$$

Here $(N_A)_j(t)$ corresponds to the number of particles of substance A in the state j at the time $t \in [0,t_f]$.

Similar remarks hold for $(N_B)_k(t)$ and $(N_C)_l(t)$.

We denoted by $\mathbf{v}(\mathbf{r}(x,t),t)$ the macroscopic fluid velocity field at position $\mathbf{r}(x,t)$ and time $t \in [0,t_f]$.

We assume the obvious constraint

$$\mathbf{v}(\mathbf{r}(x,t),t) = \frac{\partial \mathbf{r}(x,t)}{\partial t},\ \text{in } \Omega \times [0,t_f].$$

Moreover, the corresponding particle fields are denoted by

$$|(\phi_A)_j(\mathbf{r}(x,t),t)|^2,$$

$$|(\phi_B)_k(\mathbf{r}(x,t),t)|^2,$$

and

$$|(\phi_C)_l(\mathbf{r}(x,t),t)|^2$$

so that we define

$$\begin{aligned}\rho(\mathbf{r}(x,t),t) &= \sum_{j=1}^{(N_A)_0(t)} |(\phi_A)_j(\mathbf{r}(x,t),t)|^2 \\ &\quad + \sum_{k=1}^{(N_B)_0(t)} |(\phi_B)_k(\mathbf{r}(x,t),t)|^2 \\ &\quad + \sum_{l=1}^{(N_C)_0(t)} |(\phi_C)_l(\mathbf{r}(x,t),t)|^2. \end{aligned} \tag{1}$$

$$\rho_A(\mathbf{r}(x,t),t) = \sum_{j=1}^{(N_A)_0(t)} |(\phi_A)_j(\mathbf{r}(x,t),t)|^2, \tag{2}$$

$$\rho_B(\mathbf{r}(x,t),t) = \sum_{k=1}^{(N_A)_0(t)} |(\phi_B)_k(\mathbf{r}(x,t),t)|^2, \tag{3}$$

and

$$\rho_C(\mathbf{r}(x,t),t) = \sum_{l=1}^{(N_C)_0(t)} |(\phi_C)_j(\mathbf{r}(x,t),t)|^2, \text{ in } \Omega \times [0,t_f]. \tag{4}$$

Furthermore, the system is subject to the following constraints

$$\begin{aligned} m_A(t) &= (m_A)_0 - \alpha_A m_C(t) \\ &\quad -\alpha_A \int_0^t \int_{\partial\Omega} \rho_C(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\, dS\, d\tau \\ &\quad -\int_0^t \int_{\partial\Omega} \rho_A(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\, dS\, d\tau, \end{aligned} \tag{5}$$

$$\begin{aligned} m_B(t) &= (m_B)_0 - \alpha_B m_C(t) \\ &\quad -\alpha_B \int_0^t \int_{\partial\Omega} \rho_C(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\, dS\, d\tau \\ &\quad -\int_0^t \int_{\partial\Omega} \rho_B(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\, dS\, d\tau, \end{aligned} \tag{6}$$

where $\mathbf{n}$ denotes the outward normal field to $\partial\Omega = S$.

Moreover,

$$m_C(t) = \int_\Omega \rho_C(\mathbf{r}(x,t),t)\,dx,$$

$$m_A(t) = \int_\Omega \rho_A(\mathbf{r}(x,t),t)\,dx,$$

and

$$m_B(t) = \int_\Omega \rho_B(\mathbf{r}(x,t),t)\,dx.$$

Also, we assume the total mass of substance C only increases in time, so that

$$\frac{d}{dt}\left(m_C(t) + \int_0^t \int_{\partial\Omega} \rho_C(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\,dS\,d\tau\right) \geq 0, \text{ in } [0,t_f]$$

and

$$\frac{d\rho_c}{dt} + \rho_c \text{ div } \mathbf{v} \geq 0, \text{ in } \Omega \times [0,t_f].$$

Here $\hat{\mathbf{r}}$ refers to internal vibrational degrees of freedom related to the internal energy and temperature concepts.

We assume also the following constraints,

$$\begin{aligned} C_vT \;=\; & \sum_{j=1}^{(N_A)_0(t)} \frac{1}{2}|(\phi_A)_j|^2 \frac{d(\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t)}{dt}\cdot\frac{d(\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t)}{dt} \\ & + \sum_{k=1}^{(N_B)_0(t)} \frac{1}{2}|(\phi_A)_k|^2 \frac{d(\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t)}{dt}\cdot\frac{d(\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t)}{dt} \\ & + \sum_{l=1}^{(N_C)_0(t)} \frac{1}{2}|(\phi_C)_l|^2 \frac{d(\hat{\mathbf{r}}_C)_l(\mathbf{r}(x,t),t)}{dt}\cdot\frac{d(\hat{\mathbf{r}}_C)_l(\mathbf{r}(x,t),t)}{dt}, \end{aligned} \tag{7}$$

in $\Omega \times [0,t_f]$.

Moreover, following the approach in reference [16],

$$C_v\rho\frac{dT(\mathbf{r}(x,t),t)}{dt} - K\nabla^2T - \rho\mathbf{g}\cdot\mathbf{v} + P \text{ div } (\mathbf{v}) - \frac{\partial Q}{\partial t} = 0,$$

in $\Omega \times [0,t_f]$, where as previously indicated,

$$T = T(\mathbf{r}(x,t),t)$$

is the temperature scalar field, $C_v > 0$, $K > 0$ are real constants, Q is a heat function and

$$P = P(\hat{P}, \rho) = \frac{d(\hat{P}\rho)}{dt}.$$

As usual, we generically denote

$$\langle f, h\rangle_{L^2} = \int_0^{t_f} \int_\Omega f\, h\, dxdt,\ \forall f, h \in L^2(\Omega \times [0, t_f]),$$

with an analogous notation for a vectorial case.

Denoting

$$\partial\Omega = \partial\Omega_1 \cup \partial\Omega_2 \cup \partial\Omega_3 \cup \partial\Omega_4$$

where such sets $\partial\Omega_j$ are connected and such a union is quasi-disjoint, we define following spaces

$$V = \{\mathbf{r} \in W^{1,2}(\Omega \times [0, t_f]; \mathbb{R}^3) \ :\ \mathbf{r}(0, x) = \mathbf{r}_0\},$$

and generically

$$Y_1 = \{\mathbf{v} \in W^{1,2} \ :\ \mathbf{v}|_{t=0} = \mathbf{v}_0(x) \text{ and } \mathbf{v} = \mathbf{0} \text{ on } (\partial\Omega_3 \cup \partial\Omega_4) \times [0, t_f]\},$$

$$Y_2 = \{T \in W^{1,2} \ :\ T|_{t=0} = T_0(x) \text{ and } T = T_1 \text{ on } \partial\Omega \times [0, t_f]\},$$

$$\begin{aligned} Y_3 \quad = \quad & \{(\hat{\mathbf{r}}, \phi) \in W^{1,2} \ :\ (\rho_A, \rho_B, \rho_C) = ((\rho_A)_1, (\rho_B)_1, 0) \\ & \text{in } \partial\Omega_1 \times [0, t_f] \ :\ (\rho_A, \rho_B, \rho_C)|_{t=0} = ((\rho_A)_0, (\rho_B)_0, 0)\} \end{aligned} \tag{8}$$

and

$$Y_4 = L^2.$$

In order to model such a chemical reaction, we define the following functional

$$J : V \times Y_1 \times Y_2 \times Y_3 \times Y_4 \to \mathbb{R},$$

where

$$
\begin{aligned}
&J(\mathbf{r}, \mathbf{v}, T, \{(r_A)_j\}, \{(r_B)_k\}, \{(r_C)_l\}, \{(\phi_A)_j\}, \{(\phi_B)_k\}, \{(\phi_C)_l\}, \hat{P}, \lambda, E, \mu, N) \\
= \;& \frac{1}{2}\int_0^{t_f} \int_\Omega \rho(\mathbf{r}(x,t),t)\mathbf{v}(\mathbf{r}(x,t),t) \cdot \mathbf{v}(\mathbf{r}(x,t),t)\; dxdt \\
&+\frac{1}{2}\int_0^{t_f} \int_\Omega \sum_{j=1}^{(N_A)_0(t)} \frac{1}{2}|(\phi_A)_j|^2 \frac{d(\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d(\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t)}{dt}\; dxdt \\
&+\frac{1}{2}\int_0^{t_f} \int_\Omega \sum_{k=1}^{(N_B)_0(t)} \frac{1}{2}|(\phi_B)_k|^2 \frac{d(\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d(\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t)}{dt} dxdt \\
&+\frac{1}{2}\int_0^{t_f} \int_\Omega \sum_{l=1}^{(N_C)_0(t)} \frac{1}{2}|(\phi_C)_l|^2 \frac{d(\hat{\mathbf{r}}_C)_l(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d(\hat{\mathbf{r}}_C)_l(\mathbf{r}(x,t),t)}{dt}\; dxdt \\
&+\int_0^{t_f} \int_\Omega \sum_{j=1}^{(N_A)_0(t)} |(\phi_A)_j|^2 \frac{d(\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d\mathbf{r}(x,t)}{dt}\; dxdt \\
&+\int_0^{t_f} \int_\Omega \sum_{k=1}^{(N_B)_0(t)} |(\phi_B)_k|^2 \frac{d(\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d\mathbf{r}(x,t)}{dt} dxdt \\
&+\int_0^{t_f} \int_\Omega \sum_{l=1}^{(N_C)_0(t)} |(\phi_C)_l|^2 \frac{d(\hat{\mathbf{r}}_C)_l((\mathbf{r}(x,t),t)}{dt} \cdot \frac{d\mathbf{r}(x,t)}{dt}\; dxdt \\
&+\int_0^{t_f} \int_\Omega \lambda \left(\mathbf{v}(\mathbf{r}(x,t),t) - \frac{\partial \mathbf{r}(x,t)}{\partial t} \right)\; dxdt \\
&-\int_0^{t_f} \int_\Omega \hat{P} \left(\frac{d\rho}{dt} + \rho \;\mathrm{div}\; \mathbf{v} \right)\; dxdt \\
&+\int_0^{t_f} \int_\Omega \lambda_1 \left(\rho\, C_v\, \frac{d\,T}{dt} - K\nabla^2 T + P \;\mathrm{div}\; \mathbf{v} - \rho \mathbf{g} \cdot \mathbf{v} \right)\; dxdt \\
&+\int_0^{t_f} \int_\Omega \lambda_2 \left(C_v T - \sum_{j=1}^{(N_A)_0(t)} \frac{1}{2}|(\phi_A)_j|^2 \frac{d(\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d(\hat{\mathbf{r}}_A)_j(\mathbf{r}(x,t),t)}{dt} \right. \\
&- \sum_{k=1}^{(N_B)_0(t)} \frac{1}{2}|(\phi_B)_k|^2 \frac{d(\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d(\hat{\mathbf{r}}_B)_k(\mathbf{r}(x,t),t)}{dt} \\
&\left. - \sum_{l=1}^{(N_C)_0(t)} \frac{1}{2}|(\phi_C)_l|^2 \frac{d(\hat{\mathbf{r}}_C)_l(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d(\hat{\mathbf{r}}_C)_l(\mathbf{r}(x,t),t)}{dt} \right)\; dxdt \\
&+J_{Aux_1} + J_{Aux_2} + J_{Aux_3} - \int_0^{t_f} \int_\Omega T(\mathbf{r}(x,t),t) dS(x,t), \qquad (9)
\end{aligned}
$$

where here generically $\nabla\varphi(x,t) = (i\varphi_t(x,t), \varphi_{x_1}(x,t), \varphi_{x_2}(x,t), \varphi_{x_3}(x,t))$ where $i \in \mathbb{C}$ denotes the imaginary unit and

$$\begin{aligned}
J_{Aux_1} &= \frac{\gamma_A}{2}\int_0^{t_f}\sum_{j=1}^{(N_A)_0(t)}\int_\Omega \nabla(\phi_A)_j\cdot\nabla(\phi_A)_j\,dxdt \\
&+\frac{\gamma_B}{2}\int_0^{t_f}\sum_{k=1}^{(N_B)_0(t)}\int_\Omega \nabla(\phi_B)_k\cdot\nabla(\phi_B)_k\,dxdt \\
&+\frac{\gamma_C}{2}\int_0^{t_f}\sum_{l=1}^{(N_C)_0(t)}\int_\Omega \nabla(\phi_C)_l\cdot\nabla(\phi_C)_l\,dxdt \\
&+\frac{\alpha_A}{2}\int_0^{t_f}\sum_{j=1}^{(N_A)_0(t)}\int_\Omega |(\phi_A)_j|^4\,dxdt \\
&+\frac{\alpha_B}{2}\int_0^{t_f}\sum_{k=1}^{(N_B)_0(t)}\int_\Omega |(\phi_B)_k|^4\,dxdt \\
&+\frac{\alpha_C}{2}\int_0^{t_f}\sum_{j=1}^{(N_C)_0(t)}\int_\Omega |(\phi_C)_l|^4\,dxdt, \qquad (10)
\end{aligned}$$

$$\begin{aligned}
&J_{Aux_2} \\
= &\int_0^{t_f}\lambda_3(t)\Big(m_A(t)-(m_A)_0+\int_0^t\int_{\partial\Omega}\rho_A(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\,dS\,d\tau \\
&+\alpha_A m_C(t)+\alpha_A\int_0^t\int_{\partial\Omega}\rho_C(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}dS\,d\tau\Big)\,dt \\
&+\int_0^{t_f}\lambda_4(t)\Big(m_B(t)-(m_B)_0+\int_0^t\int_{\partial\Omega}\rho_B(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\,dS\,d\tau \\
&+\alpha_B m_C(t)+\alpha_B\int_0^t\int_{\partial\Omega}\rho_C(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}dS\,d\tau\Big)\,dt \\
&-\int_0^{t_f}\lambda_5(t)^2\Big(\frac{d}{dt}(m_C(t) \\
&+\int_0^t\int_{\partial\Omega}\rho_C(\mathbf{r}(x,\tau),\tau)\mathbf{v}(\mathbf{r}(x,\tau),\tau)\cdot\mathbf{n}\,dS\,d\tau)\Big)\,dt \\
&-\int_0^{t_f}\int_\Omega\lambda_6(x,t)^2\left(\frac{d\rho_c}{dt}+\rho_c\,\mathrm{div}\,\mathbf{v}\right)\,dxdt, \qquad (11)
\end{aligned}$$

and

$$
\begin{aligned}
& J_{Aux_3} \\
= \; & -\int_0^{t_f} \sum_{j=1}^{(N_A)_0(t)} E_j^A(t) \left(\int_\Omega |(\phi_A)_j|^2 \, dx - (N_A)_j(t) m_{p_A} \right) \, dt \\
& -\int_0^{t_f} \sum_{k=1}^{(N_B)_0(t)} E_k^B(t) \left(\int_\Omega |(\phi_B)_k|^2 \, dx - (N_B)_k(t) m_{p_B} \right) \, dt \\
& -\int_0^{t_f} \sum_{l=1}^{(N_C)_0(t)} E_l^C(t) \left(\int_\Omega |(\phi_C)_l|^2 \, dx - (N_C)_l(t) m_{p_C} \right) \, dt \\
& -\int_0^{t_f} \mu_7(t) \left(N_A(t) - \sum_{j=1}^{(N_A)_0(t)} (N_A)_j(t) \right) \, dt \\
& -\int_0^{t_f} \mu_8(t) \left(N_B(t) - \sum_{k=1}^{(N_B)_0(t)} (N_B)_k(t) \right) \, dt \\
& -\int_0^{t_f} \mu_9(t) \left(N_C(t) - \sum_{l=1}^{(N_C)_0(t)} (N_A)_l(t) \right) \, dt \\
& +\int_0^{t_f} \lambda_{10}(t) \left(m_A(t) - N_A(t) m_{p_A} \right) \, dt \\
& +\int_0^{t_f} \lambda_{11}(t) \left(m_B(t) - N_B(t) m_{p_B} \right) \, dt \\
& +\int_0^{t_f} \lambda_{12}(t) \left(m_C(t) - N_C(t) m_{p_C} \right) \, dt. \qquad (12)
\end{aligned}
$$

Finally, the entropy differential is defined by

$$
\begin{aligned}
dS(x,t) \;=\; & -\sum_{j=1}^{(N_A)_0(t)} \frac{(N_A)_j(t)}{N_A(t)} \ln\left(\frac{(N_A)_j(t)}{N_A(t)\; e}\right) \, dxdt \\
& -\sum_{k=1}^{(N_B)_0(t)} \frac{(N_B)_k(t)}{N_B(t)} \ln\left(\frac{(N_B)_k(t)}{N_B(t)\; e}\right) \, dxdt \\
& -\sum_{l=1}^{(N_C)_0(t)} \frac{(N_C)_l(t)}{N_C(t)} \ln\left(\frac{(N_C)_l(t)}{N_C(t)\; e}\right) \, dxdt \\
& -\frac{(N_A)(t)}{N_T(t)} \ln\left(\frac{(N_A)(t)}{N_T(t)\; e}\right) \, dxdt \\
& -\frac{(N_B)(t)}{N_T(t)} \ln\left(\frac{(N_B)(t)}{N_T(t)\; e}\right) \, dxdt \\
& -\frac{(N_C)(t)}{N_T(t)} \ln\left(\frac{(N_C)(t)}{N_T(t)\; e}\right) \, dxdt, \qquad (13)
\end{aligned}
$$

where $N_T(t) = N_A(t) + N_B(t) + N_C(t)$.

Here N_T denotes the total number of particles at the time t and $(N_A)_j(t)$ the number of particles at the state corresponding to $E_j^A(t), \forall j \in \{1, \cdots, (N_A)_0(t)\}$, in $[0, t_f]$.

Remark 2.1. *Observe that the variation of J in $(N_A)_j$ give us*

$$
-E_j^A(t)m_{p_A} - \mu_7(t) - \int_\Omega T\, dx \frac{\partial}{\partial (N_A)_j(t)} \left(\frac{(N_A)_j(t)}{N_A(t)} \ln\left(\frac{(N_A)_j(t)}{N_A(t)}\right) - \frac{(N_A)_j(t)}{N_A(t)} \right) = 0,
$$

so that

$$
-E_j^A(t)m_{p_A} - \mu_7(t) - \int_\Omega T \;\; dx \left(\frac{1}{N_A(t)} \ln\left(\frac{(N_A)_j(t)}{N_A(t)}\right) + \frac{1}{N_A(t)} - \frac{1}{N_A(t)} \right) = 0,
$$

that is,

$$
-E_j^A(t)m_{p_A} - \mu_7(t) - \int_\Omega T\, dx \frac{1}{N_A(t)} \ln\left(\frac{(N_A)_j(t)}{N_A(t)}\right) = 0,
$$

and thus

$$
\frac{(N_A)_j(t)}{N_A(t)} = e^{N_A(t)\left(\frac{-E_j^A(t)m_{p_A} - \mu_7(t)}{\int_\Omega T\, dx}\right)}, \quad \textit{in } [0, t_f].
$$

Such a distribution in $(N_A)_j(t)$ obtained is similar, in some sense, to the well known Gibbs one obtained in statistical physics.

3 A related model in superconductivity

In this section we develop a variational formulation for a model in superconductivity.

The model here addressed is an extension of the standard Ginzburg-Landau one which additionally includes an eigenvalue formulation related to a total system mass restriction. For details on the standard Ginzburg-Landau model and related ones in superconductivity, please see [17, 18, 19].

Let $\Omega \subset \mathbb{R}^3$ be an open, bounded and connected set with a regular (Lipschitzian) boundary denoted by $\partial\Omega$.

Consider an electronic field comprised by particles which may be in a normal or in a super-conducting phase.

Indeed, in this section our aim is to model a phase transition from a normal phase to a super-conducting one of a solid with a corresponding volume defined by Ω in a time interval $[0, t_f]$. Here again $t \in [0, t_f]$ denotes time.

For the normal phase, the position fields for such particles are denoted by

$$\mathbf{r}_j^N(x,t) = x + \hat{\mathbf{r}}_j^N(x,t),\ \forall j \in \{1, \cdots, N_0^N(t)\},\ \text{in } \Omega \times [0, t_f].$$

For the super-conducting phase, the particle position fields are denoted by

$$\mathbf{r}_j^S(x,t) = x + \hat{\mathbf{r}}_j^S(x,t),\ \forall j \in \{1, \cdots, N_0^S(t)\},\ \text{in } \Omega \times [0, t_f].$$

The related density fields are denoted by

$$|\phi_j^N(x,t)|^2,\ \forall j \in \{1, \cdots, N_0^N(t)\},\ \text{in } \Omega \times [0, t_f],$$

for the normal phase and

$$|\phi_j^S(x,t)|^2,\ \forall j \in \{1, \cdots, N_0^S(t)\},\ \text{in } \Omega \times [0, t_f],$$

for the super-conducting one.

The macroscopic temperature field is denoted by

$$T = T(x,t),\ \ \text{in } \Omega \times [0, t_f].$$

We assume the following constraints

$$\begin{aligned} C_v T &= \sum_{j=1}^{N_0^N(t)} \frac{1}{2} |\phi_j^N(x,t)|^2 \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t} \\ &+ \sum_{j=1}^{N_0^S(t)} \frac{1}{2} |\phi_j^S(x,t)|^2 \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t}, \end{aligned} \tag{14}$$

and following an analogous approach in reference [1 6]and neglecting the effect of the electric and magnetic fields on the temperature,

$$C_v \rho \frac{\partial T}{\partial t} - K \nabla^2 T - \frac{\partial Q}{\partial t} = 0, \text{ in } \Omega \times [0, t_f],$$

where $C_v > 0$, $K > 0$ are appropriate real constants and Q is heat function.

In these previous lines we have denoted

$$N^N(t) = \sum_{j=1}^{N_0^N(t)} N_j^N(t),$$

and

$$N^S(t) = \sum_{j=1}^{N_0^S(t)} N_j^S(t),$$

where $N_j^N(t)$ and $N_j^S(t)$ correspond to the number of particles in the states corresponding to the eigenvalues $E_j^N(t)$ and $E_j^S(t)$, for the normal and super-conducting phases, respectively.

We have also defined the macroscopic density ρ by

$$\rho(x,t) = \rho_N(x,t) + \rho_S(x,t) \equiv |\phi(x,t)|^2,$$

where

$$\rho_N(x,t) = \sum_{j=1}^{N_0^N(t)} |\phi_j^N(x,t)|^2 \equiv |\phi_N(x,t)|^2,$$

and

$$\rho_S(x,t) = \sum_{j=1}^{N_0^S(t)} |\phi_j^S(x,t)|^2 \equiv |\phi_S(x,t)|^2.$$

The system is also subject to the following mass constraint

$$\int_\Omega \rho(x,t)\, dx = m_T, \text{ in } [0, t_f],$$

where m_T is the fixed total system mass.

With such statements and definitions in mind, we define the following functional J, which corresponds to variational formulation for the model in question,

where

$$
\begin{aligned}
& J(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\},\{\mathbf{r}_j^N(\varepsilon t,x)\},\{\mathbf{r}_j^S(\varepsilon t,x)\},\{\phi_j^N\},\{\phi_j^S\},T,\mathbf{A},V,E,\lambda,N) \\
= \quad & -\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^N(t)} \frac{1}{2}|\phi_j^N(x,t)|^2 \frac{\partial \hat{\mathbf{r}}_j^N(x,t)}{\partial t}\cdot\frac{\partial \hat{\mathbf{r}}_j^N(x,t)}{\partial t}\,dxdt \\
& -\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^S(t)} \frac{1}{2}|\phi_j^S(x,t)|^2 \frac{\partial \hat{\mathbf{r}}_j^S(x,t)}{\partial t}\cdot\frac{\partial \hat{\mathbf{r}}_j^S(x,t)}{\partial t}\,dxdt \\
& +K_1\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^N(t)} |\phi_j^N(x,t)|^2 \mathbf{A}\cdot\frac{\partial \mathbf{r}_j^N(x,t)}{\partial t}\,dxdt \\
& +K_1\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^S(t)} |\phi_j^S(x,t)|^2 \mathbf{A}\cdot\frac{\partial \mathbf{r}_j^S(x,t)}{\partial t}\,dxdt \\
& +\frac{\gamma_N}{2}\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^N(t)} |\nabla\phi_j^N - iw_N\mathbf{A}\phi_j^N|^2\,dxdt \\
& +\frac{\gamma_S}{2}\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^S(t)} |\nabla\phi_j^S - iw_S\mathbf{A}\phi_j^S|^2\,dxdt \\
& -\frac{(\gamma_0)_N}{2}\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^N(t)} \frac{\partial\phi_j^N(x,t)}{\partial t}\,\frac{\partial(\phi_j^N)^*(x,t)}{\partial t}\,dxdt \\
& -\frac{(\gamma_0)_S}{2}\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^S(t)} \frac{\partial\phi_j^S(x,t)}{\partial t}\,\frac{\partial(\phi_j^S)^*(x,t)}{\partial t}\,dxdt \\
& +\frac{\alpha_N}{4}\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^N(t)} |\phi_j^N|^4\,dxdt + \frac{\alpha_S}{4}\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^S(t)} |\phi_j^S|^4\,dxdt \\
& +J_{Aux_1}+J_{Aux_2}+J_{Aux_3}, \qquad (15)
\end{aligned}
$$

where

$$
\begin{aligned}
J_{Aux_1} &= -\int_0^{t_f} \sum_{j=1}^{N_0^N(t)} E_j^N(t) \left(\int_\Omega |\phi_j^N(x,t)|^2 \, dx - N_j^N(t) m_{p_N} \right) \, dt \\
&\quad - \int_0^{t_f} \sum_{j=1}^{N_0^S(t)} E_j^S(t) \left(\int_\Omega |\phi_j^S(x,t)|^2 \, dx - N_j^S(t) m_{p_S} \right) \, dt \\
&\quad - \int_0^{t_f} \mu_N(t) \left(N_N(t) - \sum_{j=1}^{N_0^N(t)} N_j^N(t) \right) \, dt \\
&\quad - \int_0^{t_f} \mu_S(t) \left(N_S(t) - \sum_{j=1}^{N_0^S(t)} N_j^S(t) \right) \, dt \\
&\quad - \int_0^{t_f} E(t) (N_N(t) m_{p_N} + N_S(t) m_{p_S} - m_T) \, dt, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
& J_{Aux_2} \\
&= \int_0^{t_f} \int_\Omega \lambda_1 \left(C_v T - \sum_{j=1}^{N_0^N(t)} \frac{1}{2} |\phi_j^N(x,t)|^2 \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t} \right. \\
&\quad \left. - \sum_{j=1}^{N_0^S(t)} \frac{1}{2} |\phi_j^S(x,t)|^2 \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t} \right) \, dx dt \\
&\quad + \int_0^{t_f} \int_\Omega \lambda_2 \left(C_v \rho \frac{\partial T}{\partial t} - K \nabla^2 T - \frac{\partial Q}{\partial t} \right) \, dx dt, \qquad (17)
\end{aligned}
$$

and

$$\begin{aligned}
J_{Aux_3} &= -\int_0^{t_f}\int_\Omega K_1 V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})\left(\sum_{j=1}^{N_0^N(t)}|\phi_j^N|^2+\sum_{j=1}^{N_0^S(t)}|\phi_j^S|^2\right)\,dxdt\\
&+K_1\int_0^{t_f}\int_\Omega\sum_{j=1}^{N_0^N(t)}|\phi_j^N|^2\mathbf{B}(x,t)\times\frac{\partial\mathbf{r}_j^N(x,t)}{\partial t}\cdot\left(\mathbf{r}_j^N(\varepsilon t,x)-(\mathbf{r}_0)_j^N(x)\right)\,dxdt\\
&+K_1\int_0^{t_f}\int_\Omega\sum_{j=1}^{N_0^S(t)}|\phi_j^S|^2\mathbf{B}(x,t)\times\frac{\partial\mathbf{r}_j^S(x,t)}{\partial t}\cdot\left(\mathbf{r}_j^S(\varepsilon t,x)-(\mathbf{r}_0)_j^S(x)\right)\,dxdt\\
&+\frac{1}{8\pi}\int_0^{t_f}\int_\Omega|\text{ curl }\mathbf{A}-\mathbf{B}_0|^2\,dxdt\\
&+\frac{1}{8\pi}\int_0^{t_f}\int_\Omega\left|\nabla V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})+\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right|^2\,dxdt, \qquad (18)
\end{aligned}$$

where

$$\mathbf{B}=\text{ curl }\mathbf{A}-\mathbf{B}_0$$

is the total magnetic field, $\mathbf{A}=(A_1,A_2,A_3)$ is the magnetic potential,

$$V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})$$

is the electric potential and

$$\mathbf{E}=-\nabla V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},$$

is the electric field.

Remark 3.1. *As a final remark, about the domain of the functional J we have not so far addressed it explicitly. However for such a domain we would specify sets of generically smooth C^2 class functions defining only the initial conditions*

$$\{\mathbf{r}_j^N\}|_{t=0}=\{(\mathbf{r}_0)_j^N\},$$

$$\{\mathbf{r}_j^S\}|_{t=0}=\{(\mathbf{r}_0)_j^S\}$$

and $T(x,0)=T_0$, considering the remaining boundary and initial conditions just as natural ones.

3.1 A simplified macroscopic approach concerning the previous model

In this section we define another variational formulation for the model in question through a simplified approach.

Considering the statements and context of the previous section, at first we define the macroscopic position fields $\mathbf{r}_N(x,t)$ and $\mathbf{r}_S(x,t)$ for the normal and super-conducting phases, respectively.

Similarly, we define the corresponding density fields $|\phi_N(x,t)|^2$ and $|\phi_S(x,t)|^2$ also for the normal and super-conducting phases, respectively.

For the macroscopic temperature field $T = T(x,t)$, we set the following constraints

$$\begin{aligned} C_v T &= \frac{1}{2}|\phi_N(x,t)|^2 \frac{\partial \mathbf{r}_N}{\partial t} \cdot \frac{\partial r_N}{\partial t} \\ &+ \frac{1}{2}|\phi_S(x,t)|^2 \frac{\partial \mathbf{r}_S}{\partial t} \cdot \frac{\partial \mathbf{r}_S}{\partial t}, \end{aligned} \tag{19}$$

and following an analogous approach in reference [16] and neglecting the effect of the electric and magnetic fields on the temperature,

$$C_v \rho \frac{\partial T}{\partial t} - K\nabla^2 T - \frac{\partial Q}{\partial t} = 0, \text{ in } \Omega \times [0, t_f],$$

where $C_v > 0$, $K > 0$ are appropriate real constants and Q is heat function.

Moreover $\rho(x,t) = |\phi_N(x,t)|^2 + |\phi_S(x,t)|^2$, in $\Omega \times [0, t_f]$,

Following again the context of previous section, we define the following func-

tional J_3, which corresponds to a variational formulation for the model in question,

$$
\begin{aligned}
& J_3(\mathbf{r}_N, \mathbf{r}_S, \mathbf{r}_N(\varepsilon t, x), \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, \mathbf{A}, V, E) \\
= & -\int_0^{t_f} \int_\Omega |\phi_N|^2 \frac{\partial \mathbf{r}_N}{\partial t} \cdot \frac{\partial \mathbf{r}_N}{\partial t}\, dxdt \\
& -\int_0^{t_f} \int_\Omega |\phi_S|^2 \frac{\partial \mathbf{r}_S}{\partial t} \cdot \frac{\partial \mathbf{r}_S}{\partial t}\, dxdt \\
& +K_1 \int_0^{t_f} \int_\Omega |\phi_N(x,t)|^2 \mathbf{A} \cdot \frac{\partial \mathbf{r}_N(x,t)}{\partial t}\, dxdt \\
& +K_1 \int_0^{t_f} \int_\Omega |\phi_S(x,t)|^2 \mathbf{A} \cdot \frac{\partial \mathbf{r}_S(x,t)}{\partial t}\, dxdt \\
& +\frac{\gamma_N}{2} \int_0^{t_f} \int_\Omega |\nabla \phi_N - i w_N \mathbf{A} \phi_N|^2\, dxdt \\
& +\frac{\gamma_S}{2} \int_0^{t_f} \int_\Omega |\nabla \phi_S - i w_S \mathbf{A} \phi_S|^2\, dxdt \\
& -\frac{(\gamma_0)_N}{2} \int_0^{t_f} \int_\Omega \frac{\partial \phi_N(x,t)}{\partial t} \frac{\partial (\phi_N)^*(x,t)}{\partial t}\, dxdt \\
& -\frac{(\gamma_0)_S}{2} \int_0^{t_f} \int_\Omega \frac{\partial \phi_S(x,t)}{\partial t} \frac{\partial (\phi_S)^*(x,t)}{\partial t}\, dxdt \\
& +\frac{\alpha_N}{4} \int_0^{t_f} \int_\Omega |\phi_N|^4\, dxdt + \frac{\alpha_S}{4} \int_0^{t_f} \int_\Omega |\phi_S|^4\, dxdt \\
& +J_{Aux_1} + J_{Aux_2} + J_{Aux_3},
\end{aligned} \tag{20}
$$

where

$$
J_{Aux_1} = -\int_0^{t_f} E(t) \left(\int_\Omega (|\phi_N|^2 + |\phi_S|^2)\, dx - m_T \right)\, dt,
$$

$$
\begin{aligned}
J_{Aux_2} = & \int_0^{t_f} \int_\Omega \lambda_1 \left(C_v T - \frac{1}{2} |\phi_N(x,t)|^2 \frac{\partial \mathbf{r}_N}{\partial t} \cdot \frac{\partial r_N}{\partial t} \right. \\
& \left. -\frac{1}{2} |\phi_S(x,t)|^2 \frac{\partial \mathbf{r}_S}{\partial t} \cdot \frac{\partial \mathbf{r}_S}{\partial t} \right)\, dxdt \\
& + \int_0^{t_f} \int_\Omega \lambda_2 \left(\rho C_v \frac{\partial T}{\partial t} - K \nabla^2 T - \frac{\partial Q}{\partial t} \right)\, dxdt
\end{aligned} \tag{21}
$$

and

$$
\begin{aligned}
J_{Aux_3} \;=\; & -K_1 \int_0^{t_f} \int_\Omega V(\mathbf{r}_N, \mathbf{r}_S) \left(|\phi_N|^2 + |\phi_S|^2 \right) \, dxdt \\
& + K_1 \int_0^{t_f} \int_\Omega |\phi_N|^2 \mathbf{B}(x,t) \times \frac{\partial \mathbf{r}_N(x,t)}{\partial t} \cdot \left(\mathbf{r}_N(\varepsilon t, x) - (\mathbf{r}_0)_N(x) \right) \, dxdt \\
& + K_1 \int_0^{t_f} \int_\Omega |\phi_S|^2 \mathbf{B}(x,t) \times \frac{\partial \mathbf{r}_S(x,t)}{\partial t} \cdot \left(\mathbf{r}_S(\varepsilon t, x) - (\mathbf{r}_0)_S(x) \right) \, dxdt \\
& + \frac{1}{8\pi} \int_0^{t_f} \int_\Omega | \text{ curl } \mathbf{A} - \mathbf{B}_0 |^2 \, dxdt \\
& + \frac{1}{8\pi} \int_0^{t_f} \int_\Omega \left| \nabla V(\mathbf{r}_N, \mathbf{r}_S) + \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} \right|^2 \, dxdt, \qquad (22)
\end{aligned}
$$

where

$$\mathbf{B} = \text{ curl } \mathbf{A} - \mathbf{B}_0$$

is the total magnetic field, $\mathbf{A} = (A_1, A_2, A_3)$ is the magnetic potential,

$$V(\mathbf{r}_N, \mathbf{r}_S)$$

is the electric potential and

$$\mathbf{E} = -\nabla V(\mathbf{r}_N, \mathbf{r}_S) - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t},$$

is the electric field.

In an even simpler approach, here we assume there exists $\mathbf{C}_1 \in \mathbb{R}^3$ such that, qualitatively, we have

$$\mathbf{r}_N(x,t) \approx e^{i\omega_N(T)t} \mathbf{C}_1, \text{ in } \Omega \times [0, t_f],$$

so that

$$\frac{\partial \mathbf{r}_N(x,t)}{\partial t} \cdot \frac{\partial (\mathbf{r}_N)^*(x,t)}{\partial t} = \omega_N(T)^2 |\mathbf{C}_1|^2 = \beta_N(T), \text{ in } \Omega \times [0, t_f].$$

Thus

$$
\begin{aligned}
& |\phi_N|^2 \frac{\partial \mathbf{r}_N(x,t)}{\partial t} \cdot \frac{\partial (\mathbf{r}_N)^*(x,t)}{\partial t} \\
\approx \;& \beta_N(T) |\phi_N|^2. \qquad (23)
\end{aligned}
$$

With similar assumptions, we may obtain

$$
\begin{aligned}
& |\phi_S|^2 \frac{\partial \mathbf{r}_S(x,t)}{\partial t} \cdot \frac{\partial (\mathbf{r}_S)^*(x,t)}{\partial t} \\
\approx \;& \beta_S(T) |\phi_S|^2. \qquad (24)
\end{aligned}
$$

With such statements and definitions in mind, considering T a fixed real constant and $Q = 0$, we define the following functional J_1, which corresponds to a macroscopic variational formulation for the model in question

$$\begin{aligned}
& J_1(\mathbf{r}_N, \mathbf{r}_S, \mathbf{r}_N(\varepsilon t, x), \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, \mathbf{A}, V, E) \\
= \; & K_1 \int_0^{t_f} \int_\Omega |\phi_N(x,t)|^2 \mathbf{A} \cdot \frac{\partial \mathbf{r}_N(x,t)}{\partial t} \, dxdt \\
& + K_1 \int_0^{t_f} \int_\Omega |\phi_S(x,t)|^2 \mathbf{A} \cdot \frac{\partial \mathbf{r}_S(x,t)}{\partial t} \, dxdt \\
& + \frac{\gamma_N}{2} \int_0^{t_f} \int_\Omega |\nabla \phi_N - i w_N \mathbf{A} \phi_N|^2 \, dxdt \\
& + \frac{\gamma_S}{2} \int_0^{t_f} \int_\Omega |\nabla \phi_S - i w_S \mathbf{A} \phi_S|^2 \, dxdt \\
& - \frac{(\gamma_0)_N}{2} \int_0^{t_f} \int_\Omega \frac{\partial \phi_N(x,t)}{\partial t} \frac{\partial (\phi_N)^*(x,t)}{\partial t} \, dxdt \\
& - \frac{(\gamma_0)_S}{2} \int_0^{t_f} \int_\Omega \frac{\partial \phi_S(x,t)}{\partial t} \frac{\partial (\phi_S)^*(x,t)}{\partial t} \, dxdt \\
& + \frac{\alpha_N}{4} \int_0^{t_f} \int_\Omega |\phi_N|^4 \, dxdt + \frac{\alpha_S}{4} \int_0^{t_f} \int_\Omega |\phi_S|^4 \, dxdt \\
& - \frac{1}{2} \int_0^{t_f} \int_\Omega \beta_N(T) |\phi_N|^2 \, dxdt - \frac{1}{2} \int_0^{t_f} \int_\Omega \beta_S(T) |\phi_S|^2 \, dxdt \\
& + J_{Aux_1} + J_{Aux_2} + J_{Aux_3},
\end{aligned} \tag{25}$$

where

$$J_{Aux_1} = -\int_0^{t_f} E(t) \left(\int_\Omega (|\phi_N|^2 + |\phi_S|^2) \, dx - m_T \right) \, dt,$$

$$\begin{aligned}
J_{Aux_2} \; = \; & \int_0^{t_f} \int_\Omega \lambda_1 \left(\beta_N(T) - \frac{\partial \mathbf{r}_N(x,t)}{\partial t} \cdot \frac{\partial \mathbf{r}_N(x,t)}{\partial t} \right) \, dxdt \\
& + \int_0^{t_f} \int_\Omega \lambda_2 \left(\beta_S(T) - \frac{\partial \mathbf{r}_S(x,t)}{\partial t} \cdot \frac{\partial \mathbf{r}_S(x,t)}{\partial t} \right) \, dxdt,
\end{aligned} \tag{26}$$

$$\begin{aligned}
J_{Aux3} &= -K_1 \int_0^{t_f} \int_\Omega V(\mathbf{r}_N, \mathbf{r}_S) \left(|\phi_N|^2 + |\phi_S|^2\right) \, dxdt \\
&+K_1 \int_0^{t_f} \int_\Omega |\phi_N|^2 \mathbf{B}(x,t) \times \frac{\partial \mathbf{r}_N(x,t)}{\partial t} \cdot (\mathbf{r}_N(\varepsilon t, x) - (\mathbf{r}_0)_N(x)) \, dxdt \\
&+K_1 \int_0^{t_f} \int_\Omega |\phi_S|^2 \mathbf{B}(x,t) \times \frac{\partial \mathbf{r}_S(x,t)}{\partial t} \cdot (\mathbf{r}_S(\varepsilon t, x) - (\mathbf{r}_0)_S(x)) \, dxdt \\
&+\frac{1}{8\pi} \int_0^{t_f} \int_\Omega |\text{ curl } \mathbf{A} - \mathbf{B}_0|^2 \, dxdt \\
&+\frac{1}{8\pi} \int_0^{t_f} \int_\Omega \left| \nabla V(\mathbf{r}_N, \mathbf{r}_S) + \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} \right|^2 \, dxdt, \qquad (27)
\end{aligned}$$

where

$$\mathbf{B} = \text{ curl } \mathbf{A} - \mathbf{B}_0$$

is the total magnetic field, $\mathbf{A} = (A_1, A_2, A_3)$ is the magnetic potential,

$$V(\mathbf{r}_N, \mathbf{r}_S)$$

is the electric potential and

$$\mathbf{E} = -\nabla V(\mathbf{r}_N, \mathbf{r}_S) - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t},$$

is the electric field.

Remark 3.2. *The variation of J_3 in $\mathbf{r}_N$ which lead us to the correct Euler-Lagrange equation is defined by*

$$\begin{aligned}
& \hat{\delta}(J_3)_{\mathbf{r}_N}(\mathbf{r}_N, \mathbf{r}_S, \mathbf{r}_N(\varepsilon t, x), \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, \mathbf{A}, V, E; \varphi) \\
= & \lim_{h \to 0} [J_3(\mathbf{r}_N + h\varphi, \mathbf{r}_S, \mathbf{r}_N(\varepsilon t, x) + h\varphi, \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, \mathbf{A}, V, E) \\
& -J_3(\mathbf{r}_N, \mathbf{r}_S, \mathbf{r}_N(\varepsilon t, x), \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, \mathbf{A}, V, E)] / h \\
= & \mathbf{0}, \ \forall \varphi \in C_c^\infty(\Omega \times [0, t_f]; \mathbb{R}^3). \qquad (28)
\end{aligned}$$

The corresponding Euler-Lagrange equation stands for

$$\begin{aligned}
& \frac{\partial}{\partial t}\left(|\phi_N|^2 \frac{\partial \mathbf{r}_N(x,t)}{\partial t}\right) - K_1 \frac{\partial}{\partial t}\left(|\phi_N|^2 \mathbf{A}\right) \\
& K_1 \frac{\partial V}{\partial \mathbf{r}_N}(|\phi_N|^2 + |\phi_S|^2) + K_1 |\phi_N|^2 \mathbf{B} \times \frac{\partial \mathbf{r}_N}{\partial t} + \mathcal{O}(\varepsilon) \\
& -\frac{1}{4\pi} \, div \, \mathbf{E} \frac{\partial V}{\partial \mathbf{r}_N} = \mathbf{0}. \qquad (29)
\end{aligned}$$

Considering a Maxwell equation in question and letting $\varepsilon \to 0$, we obtain

$$\frac{\partial}{\partial t}\left(|\phi_N|^2\frac{\partial \mathbf{r}_N(x,t)}{\partial t}\right) - K_1\frac{\partial}{\partial t}\left(|\phi_N|^2\mathbf{A}\right)$$
$$+K_1|\phi_N|^2\mathbf{B}\times\frac{\partial \mathbf{r}_N}{\partial t} = \mathbf{0}. \tag{30}$$

We highlight the solution of the concerned system including such an Euler-Lagrange equation is computable, through the Newton's method for example, by expanding J_3 as a quadratic approximate function in φ about $(\mathbf{r}, \mathbf{r}(\varepsilon t, x))$ in each iteration.

A similar remark is valid for the variation of J_3 in $\mathbf{r}_S$ and for the functionals J and J_1.

Remark 3.3. *The variation of J_3 in $\mathbf{A}$ stands for*

$$\begin{aligned} & -\ curl\ (\ curl\ \mathbf{A}-\mathbf{B}_0) - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla V(\mathbf{r}_N,\mathbf{r}_S) + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right) \\ & +\mathcal{O}(\varepsilon) + 4\pi K_1|\phi_N|^2\frac{\partial \mathbf{r}_N}{\partial t} + +4\pi K_1|\phi_S|^2\frac{\partial \mathbf{r}_S}{\partial t} \\ = & \ \mathbf{0}. \end{aligned} \tag{31}$$

Letting $\varepsilon \to 0$, we have the following Maxwell equation

$$-\ curl\ \mathbf{B} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + 4\pi\tilde{J} = \mathbf{0},$$

where

$$\tilde{J} = K_1|\phi_N|^2\frac{\partial \mathbf{r}_N}{\partial t} + +K_1|\phi_S|^2\frac{\partial \mathbf{r}_S}{\partial t}.$$

The variation of J_3 in V stands for

$$-4\pi K_1(|\phi_N|^2 + |\phi_S|^2) -\ div\ \left(\nabla V(\mathbf{r}_N,\mathbf{r}_S) + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right) = \mathbf{0},$$

so that we obtain another Maxwell equation

$$div\ \mathbf{E} = 4\pi K_1(|\phi_N|^2 + |\phi_S|^2).$$

Moreover, from

$$\mathbf{B} =\ curl\ \mathbf{A} - \mathbf{B}_0$$

and assuming $div\ (\mathbf{B}_0) = 0$, we have got

$$div\ \mathbf{B} =\ div\ (\ curl\ \mathbf{A}) = 0,$$

which is a third Maxwell equation.

Finally, the fourth Maxwell equation is obtained recalling that

$$\mathbf{E} = -\nabla V(\mathbf{r}_N, \mathbf{r}_S) - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t},$$

so that

$$curl\ \mathbf{E} = -\ curl\ (\nabla V) - \frac{1}{c}\ curl\ \left(\frac{\partial \mathbf{A}}{\partial t}\right),$$

and therefore,

$$curl\ \mathbf{E} + \frac{1}{c}\left(\frac{\partial\ curl\ \mathbf{A}}{\partial t}\right) = \mathbf{0},$$

that is,

$$curl\ \mathbf{E} + \frac{1}{c}\left(\frac{\partial(\mathbf{B} + \mathbf{B}_0)}{\partial t}\right) = \mathbf{0}.$$

Thus, if $\mathbf{B}_0$ *is time independent, that is,*

$$\frac{\partial \mathbf{B}_0}{\partial t} = \mathbf{0},$$

we have

$$curl\ \mathbf{E} + \frac{1}{c}\left(\frac{\partial \mathbf{B}}{\partial t}\right) = \mathbf{0}.$$

In summary, we have got all the four Maxwell equations as necessary conditions for a extremal point of J_3.

Remark 3.4. *For the domains of the functionals* J_3 *and* J_1 *we would specify generically smooth* C^2 *class functions with the initial conditions*

$$\mathbf{r}_N(x,0) = (\mathbf{r}_0)_N(x),$$

$$\mathbf{r}_S(x,0) = (\mathbf{r}_0)_S(x).$$

Specifically for J_1, *we consider a fixed temperature* T *which the gradual decreasing is expected to lead the system to a transition from a normal phase to a superconducting one. This transition is in fact obtained if we have* $\alpha_N \gg \alpha_S$ *and starting with a relatively large* $\beta_N(T)$ *and small* $\beta_S T)$ *and gradually decrease the temperature, which corresponds to decrease* $\beta_N(T)$, *till obtaining a critical temperature* T_C *for which the transition is started.*

4 A more general and detailed model in superconductivity

In this section we develop a variational formulation for a more general model in superconductivity.

The model here addressed includes internal variables corresponding to vibrational degrees of freedom related to the concepts of internal energy and temperature.

Let $\Omega \subset \mathbb{R}^3$ be an open, bounded and connected set with a regular (Lipschitzian) boundary denoted by $\partial\Omega$.

Consider an electronic field comprised by particles which may be in a normal or in a super-conducting phase.

Indeed, in this section our aim is to model a phase transition from a normal phase to a super-conducting one of a solid with a corresponding volume defined by Ω in a time interval $[0, t_f]$. Here again $t \in [0, t_f]$ denotes time.

For the normal phase, the position fields for such particles are denoted by

$$\mathbf{r}_j^N(\mathbf{r}_N(x,t),t) = \mathbf{r}_N(x,t) + \hat{\mathbf{r}}_j^N(\mathbf{r}_N(x,t),t),\ \forall j \in \{1, \cdots, N_0^N(t)\},\ \text{in } \Omega \times [0, t_f].$$

For the super-conducting phase, the particle position fields are denoted by

$$\mathbf{r}_j^S(\mathbf{r}_S(x,t),t) = \mathbf{r}_S(x,t) + \hat{\mathbf{r}}_j^S(\mathbf{r}_S(x,t),t),\ \forall j \in \{1, \cdots, N_0^S(t)\},\ \text{in } \Omega \times [0, t_f].$$

Here $\hat{\mathbf{r}}_j^N$ and $\hat{\mathbf{r}}_j^S$ refers to internal energy variables related to the concept of temperature.

The related density fields are denoted by

$$|\phi_j^N(\mathbf{r}(x,t),t)|^2,\ \forall j \in \{1, \cdots, N_0^N(t)\},\ \text{in } \Omega \times [0, t_f],$$

for the normal phase and

$$|\phi_j^S(\mathbf{r}_S(x,t)|^2,\ \forall j \in \{1, \cdots, N_0^S(t)\},\ \text{in } \Omega \times [0, t_f],$$

for the super-conducting one.

The macroscopic temperature field is denoted by

$$T = T(x,t) = T_N(\mathbf{r}_N(x,t),t) + T_S(\mathbf{r}_S(x,t),t),\quad \text{in } \Omega \times [0, t_f].$$

We assume the following constraints

$$C_{v_N} T_N \quad = \quad \sum_{j=1}^{N_0^N(t)} \frac{1}{2} |\phi_j^N(\mathbf{r}_N(x,t),t)|^2 \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t}, \tag{32}$$

$$C_{v_S} T_S = \sum_{j=1}^{N_0^S(t)} \frac{1}{2} |\phi_j^S(\mathbf{r}_S(x,t),t)|^2 \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t}, \tag{33}$$

and following an analogous approach in reference [16] and neglecting the effect of the electric and magnetic fields on the temperature,

$$\begin{aligned} & C_{v_N} \rho_N \frac{dT_N}{dt} + C_{v_S} \rho_S \frac{dT_S}{dt} + P_S \ \text{div}\ (\mathbf{v}_N) \\ & + P_S \ \text{div}\ (\mathbf{v}_S) - K_N \nabla^2 T_N - K_S \nabla^2 T_S - \frac{\partial Q}{\partial t} = 0, \end{aligned} \tag{34}$$

in $\Omega \times [0, t_f]$, where $C_{v_N} > 0$, C_{v_S}, $K_N > 0$ and K_S are appropriate real constants and Q is heat function.

Moreover,

$$P = P(\hat{P}, \rho) = \frac{d(\hat{P}\rho)}{dt},$$

$$P_N = \frac{d(\hat{P}\rho_N)}{dt}$$

and

$$P_S = \frac{d(\hat{P}\rho_S)}{dt}.$$

In these previous lines we have denoted

$$N^N(t) = \sum_{j=1}^{N_0^N(t)} N_j^N(t),$$

and

$$N^S(t) = \sum_{j=1}^{N_0^S(t)} N_j^S(t),$$

where $N_j^N(t)$ and $N_j^S(t)$ correspond to the number of particles in the states corresponding to the eigenvalues $E_j^N(t)$ and $E_j^S(t)$, for the normal and super-conducting phases, respectively.

We have also defined the macroscopic density ρ by

$$\rho(x,t) = \rho_N(x,t) + \rho_S(x,t) \equiv |\phi(x,t)|^2,$$

where

$$\rho_N(\mathbf{r}_N(x,t),t)) = \sum_{j=1}^{N_0^N(t)} |\phi_j^N(\mathbf{r}_N(x,t),t)|^2 \equiv |\phi_N(\mathbf{r}_N(x,t),t)|^2,$$

and

$$\rho_S(\mathbf{r}_S(x,t),t) = \sum_{j=1}^{N_0^S(t)} |\phi_j^S(\mathbf{r}_S(x,t),t)|^2 \equiv |\phi_S(\mathbf{r}_S(x,t),t)|^2.$$

Moreover, we set the following mass conservation equation as a constraint

$$\frac{d\rho}{dt} + \rho_N \text{ div } \mathbf{v}_N + \rho_S \text{ div } \mathbf{v}_S = 0, \text{ in } \Omega \times [0, t_f],$$

where the velocity fields $\mathbf{v}_N$ and $\mathbf{v}_S$ are subject to the obvious constraints

$$\mathbf{v}_N(\mathbf{r}_N(x,t),t) = \frac{\partial \mathbf{r}_N(x,t)}{\partial t},$$

and

$$\mathbf{v}_S(\mathbf{r}_S(x,t),t) = \frac{\partial \mathbf{r}_S(x,t)}{\partial t}.$$

The system is also subject to the following mass constraint

$$\int_\Omega \rho(x,t)\, dx = m_T, \text{ in } [0, t_f],$$

where m_T is the fixed total system mass.

With such statements and definitions in mind, we define the following functional J_5, which corresponds to variational formulation for the model in question,

where

$$
\begin{aligned}
& J_5(\mathbf{r}_N, \mathbf{r}_S, \{\hat{\mathbf{r}}_j^N\}, \{\hat{\mathbf{r}}_j^S\}, \{\mathbf{r}_j^N(\varepsilon t, x)\}, \{\mathbf{r}_j^S(\varepsilon t, x)\}, \{\phi_j^N\}, \{\phi_j^S\}, T, \mathbf{A}, V, E, \lambda, N) \\
= \; & -\int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^N(t)} \frac{1}{2} |\phi_j^N(\mathbf{r}_N(x,t),t)|^2 \frac{\partial \mathbf{r}_j^N(x,t)}{\partial t} \cdot \frac{\partial \mathbf{r}_j^N(x,t)}{\partial t} \, dxdt \\
& -\int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^S(t)} \frac{1}{2} |\phi_j^S(\mathbf{r}_S(x,t),t)|^2 \frac{\partial \mathbf{r}_j^S(x,t)}{\partial t} \cdot \frac{\partial \mathbf{r}_j^S(x,t)}{\partial t} \, dxdt \\
& +K_1 \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^N(t)} |\phi_j^N|^2 \mathbf{A} \cdot \frac{\partial \mathbf{r}_j^N(x,t)}{\partial t} \, dxdt \\
& +K_1 \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^S(t)} |\phi_j^S|^2 \mathbf{A} \cdot \frac{\partial \mathbf{r}_j^S(x,t)}{\partial t} \, dxdt \\
& +\frac{\gamma_N}{2} \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^N(t)} |\nabla \phi_j^N - i w_N \mathbf{A} \phi_j^N|^2 \, dxdt \\
& +\frac{\gamma_S}{2} \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^S(t)} |\nabla \phi_j^S - i w_S \mathbf{A} \phi_j^S|^2 \, dxdt \\
& -\frac{(\gamma_0)_N}{2} \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^N(t)} \frac{\partial \phi_j^N(x,t)}{\partial t} \frac{\partial (\phi_j^N)^*(x,t)}{\partial t} \, dxdt \\
& -\frac{(\gamma_0)_S}{2} \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^S(t)} \frac{\partial \phi_j^S(x,t)}{\partial t} \frac{\partial (\phi_j^S)^*(x,t)}{\partial t} \, dxdt \\
& +\frac{\alpha_N}{4} \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^N(t)} |\phi_j^N|^4 \, dxdt + \frac{\alpha_S}{4} \int_0^{t_f} \int_\Omega \sum_{j=1}^{N_0^S(t)} |\phi_j^S|^4 \, dxdt \\
& +J_{Aux_1} + J_{Aux_2} + J_{Aux_3} + J_{Aux_4},
\end{aligned} \tag{35}
$$

where

$$
\begin{aligned}
J_{Aux_1} \; = \; & \int_0^{t_f} \int_\Omega \lambda_1 \left(\mathbf{v}_N - \frac{\partial \mathbf{r}_N(x,t)}{\partial t} \right) \, dxdt \\
& + \int_0^{t_f} \int_\Omega \lambda_2 \left(\mathbf{v}_S - \frac{\partial \mathbf{r}_S(x,t)}{\partial t} \right) \, dxdt \\
& + \int_0^{t_f} \int_\Omega \hat{P} \left(\frac{d\rho}{dt} + \rho_N \text{ div } \mathbf{v}_N + \rho_S \text{ div } \mathbf{v}_S \right) \, dxdt,
\end{aligned} \tag{36}
$$

$$\begin{aligned}
J_{Aux_2} \quad = \quad & -\int_0^{t_f} \sum_{j=1}^{N_0^N(t)} E_j^N(t) \left(\int_\Omega |\phi_j^N(\mathbf{r}_N(x,t),t)|^2 \, dx - N_j^N(t) m_{p_N} \right) \, dt \\
& -\int_0^{t_f} \sum_{j=1}^{N_0^S(t)} E_j^S(t) \left(\int_\Omega |\phi_j^S(\mathbf{r}_S(x,t),t)|^2 \, dx - N_j^S(t) m_{p_S} \right) \, dt \\
& -\int_0^{t_f} \mu_N(t) \left(N_N(t) - \sum_{j=1}^{N_0^N(t)} N_j^N(t) \right) \, dt \\
& -\int_0^{t_f} \mu_S(t) \left(N_S(t) - \sum_{j=1}^{N_0^S(t)} N_j^S(t) \right) \, dt \\
& -\int_0^{t_f} E(t)(N_N(t) m_{p_N} + N_S(t) m_{p_S} - m_T) \, dt, \qquad (37)
\end{aligned}$$

$$\begin{aligned}
& J_{Aux_3} \\
= \quad & \int_0^{t_f} \int_\Omega \lambda_3 \left(C_{v_N} \rho_N \frac{dT_N}{dt} + C_{v_S} \rho_S \frac{dT_S}{dt} + P_N \ \operatorname{div}\,(\mathbf{v}_N) + P_S \ \operatorname{div}\,(\mathbf{v}_S) \right. \\
& \left. -K_N \nabla^2 T_N - K_S \nabla^2 T_S - \frac{\partial Q}{\partial t} \right) \, dxdt \\
& + \int_0^{t_f} \int_\Omega \lambda_4 \left(C_{v_N} T_N - \sum_{j=1}^{N_0^N(t)} \frac{1}{2} |\phi_j^N(\mathbf{r}_N(x,t),t)|^2 \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^N}{\partial t} \right) \, dxdt \\
& + \int_0^{t_f} \int_\Omega \lambda_5 \left(C_{v_S} T_S - \sum_{j=1}^{N_0^S(t)} \frac{1}{2} |\phi_j^S(\mathbf{r}_S(x,t),t)|^2 \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t} \cdot \frac{\partial \hat{\mathbf{r}}_j^S}{\partial t} \right) \, dxdt, \qquad (38)
\end{aligned}$$

and

$$\begin{aligned}
J_{Aux_4} &= -\int_0^{t_f}\int_\Omega K_1 V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})\left(\sum_{j=1}^{N_0^N(t)}|\phi_j^N|^2+\sum_{j=1}^{N_0^S(t)}|\phi_j^S|^2\right)\,dxdt \\
&+K_1\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^N(t)}|\phi_j^N|^2\mathbf{B}(x,t)\times\frac{\partial\mathbf{r}_j^N(x,t)}{\partial t}\cdot\left(\mathbf{r}_j^N(\varepsilon t,x)-(\mathbf{r}_0)_j^N(x)\right)\,dxdt \\
&+K_1\int_0^{t_f}\int_\Omega \sum_{j=1}^{N_0^S(t)}|\phi_j^S|^2\mathbf{B}(x,t)\times\frac{\partial\mathbf{r}_j^S(x,t)}{\partial t}\cdot\left(\mathbf{r}_j^S(\varepsilon t,x)-(\mathbf{r}_0)_j^S(x)\right)\,dxdt \\
&+\frac{1}{8\pi}\int_0^{t_f}\int_\Omega|\text{ curl }\mathbf{A}-\mathbf{B}_0|^2\,dxdt \\
&+\frac{1}{8\pi}\int_0^{t_f}\int_\Omega\left|\nabla V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})+\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right|^2\,dxdt, \qquad (39)
\end{aligned}$$

where

$$\mathbf{B}=\text{ curl }\mathbf{A}-\mathbf{B}_0$$

is the total magnetic field, $\mathbf{A}=(A_1,A_2,A_3)$ is the magnetic potential,

$$V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})$$

is the electric potential and

$$\mathbf{E}=-\nabla V(\{\mathbf{r}_j^N\},\{\mathbf{r}_j^S\})-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},$$

is the electric field.

Remark 4.1. *About the domain of the functional J_5, we have not so far addressed it explicitly. However for such a domain we would specify sets of generically smooth C^2 class functions defining only the initial conditions*

$$\{\mathbf{r}_j^N\}|_{t=0}=\{(\mathbf{r}_0)_j^N\},$$

$$\{\mathbf{r}_j^S\}|_{t=0}=\{(\mathbf{r}_0)_j^S\}$$

and $T(x,0)=T_0$, considering the remaining boundary and initial conditions just as natural ones.

4.1 A simplified macroscopic approach for this previous model

In this section we define another variational formulation for the model in question through a simplified approach.

Considering the statements and context of the previous section, at first we define the macroscopic position fields $\mathbf{r}_N(x,t)$ and $\mathbf{r}_S(x,t)$ for the normal and super-conducting phases, respectively.

Similarly, we define the corresponding density fields $|\phi_N(\mathbf{r}(x,t),t)|^2$ and $|\phi_S(\mathbf{r}(x,t),t)|^2$ also for the normal and super-conducting phases, respectively.

Considering the inclusion of variables related to a internal energy context, we define the total fields of position, where

$$\mathbf{r}_T^N(\mathbf{r}_N(x,t),t) = \mathbf{r}_N(x,t) + \hat{\mathbf{r}}_N(\mathbf{r}_N(x,t),t),$$

and

$$\mathbf{r}_T^S(\mathbf{r}_S(x,t),t) = \mathbf{r}_S(x,t) + \hat{\mathbf{r}}_S(\mathbf{r}_S(x,t),t),$$

for the normal and super-conducting phases, respectively.

For the macroscopic temperature field

$$T = T(x,t) = T_N(\mathbf{r}_N(x,t),t) + T_S(\mathbf{r}_S(x,t),t),$$

we set the following constraints

$$C_{v_N} T_N = \frac{1}{2}|\phi_N(\mathbf{r}_N(x,t),t)|^2 \frac{\partial \mathbf{r}_N}{\partial t} \cdot \frac{\partial r_N}{\partial t}, \tag{40}$$

and

$$C_{v_S} T_S = \frac{1}{2}|\phi_S(\mathbf{r}_S(x,t),t)|^2 \frac{\partial \mathbf{r}_S}{\partial t} \cdot \frac{\partial \mathbf{r}_S}{\partial t}, \tag{41}$$

and following an analogous approach in reference [16] and neglecting the effect of the electric and magnetic fields on the temperature,

$$\begin{aligned} & C_{v_N}\rho_N \frac{dT_N}{dt} + C_{v_S}\rho_S \frac{dT_S}{dt} + P_N \ \operatorname{div}(\mathbf{v}_N) \\ & + P_S \ \operatorname{div}(\mathbf{v}_S) - K_N \nabla^2 T_N - K_S \nabla^2 T_S - \frac{\partial Q}{\partial t} = 0, \end{aligned} \tag{42}$$

in $\Omega \times [0, t_f]$, where $C_{v_N} > 0$, C_{v_S}, $K_N > 0$ and K_S are appropriate real constants and Q is heat function.

Moreover,

$$P = P(\hat{P}, \rho) = \frac{d(\hat{P}\rho)}{dt},$$

$$P_N = \frac{d(\hat{P}\rho_N)}{dt}$$

and

$$P_S = \frac{d(\hat{P}\rho_S)}{dt}.$$

We have also defined the macroscopic density ρ by

$$\rho(x,t) = \rho_N(\mathbf{r}_N(x,t),t)) + \rho_S(\mathbf{r}_S(x,t),t),$$

where

$$\rho_N(\mathbf{r}_N(x,t),t)) = |\phi_N(\mathbf{r}_N(x,t),t)|^2 \equiv |\phi_N(\mathbf{r}_N(x,t),t)|^2,$$

and

$$\rho_S(\mathbf{r}_S(x,t),t) = |\phi_S(\mathbf{r}_S(x,t),t)|^2 \equiv |\phi_S(\mathbf{r}_S(x,t),t)|^2.$$

Furthermore, we set the following mass conservation equation as a constraint

$$\frac{d\rho}{dt} + \rho_N \text{ div } \mathbf{v}_N + \rho_S \text{ div } \mathbf{v}_S = 0, \text{ in } \Omega \times [0,t_f],$$

where the velocity fields $\mathbf{v}_N$ and $\mathbf{v}_S$ are subject to the obvious constraints

$$\mathbf{v}_N(\mathbf{r}_N(x,t),t) = \frac{\partial \mathbf{r}_N(x,t)}{\partial t},$$

and

$$\mathbf{v}_S(\mathbf{r}_S(x,t),t) = \frac{\partial \mathbf{r}_S(x,t)}{\partial t}.$$

The system is also subject to the following mass constraint

$$\int_\Omega \rho(x,t)\, dx = m_T, \text{ in } [0,t_f],$$

where m_T is the fixed total system mass.

With such statements and definitions in mind, we define the following functional J_7, which corresponds to variational formulation for the model in question,

where

$$
\begin{aligned}
& J_7(\mathbf{r}_N,\mathbf{r}_S,\hat{\mathbf{r}}_N,\hat{\mathbf{r}}_S,\mathbf{r}_N(\varepsilon t,x),\mathbf{r}_S(\varepsilon t,x),\phi_N,\phi_S,T,\hat{P},\mathbf{A},V,E,\lambda) \\
= \; & -\int_0^{t_f}\int_\Omega \frac{1}{2}|\phi_N(\mathbf{r}_N(x,t),t)|^2 \frac{\partial \mathbf{r}_T^N}{\partial t}\cdot\frac{\partial \mathbf{r}_T^N}{\partial t}\,dxdt \\
& -\int_0^{t_f} \frac{1}{2}|\phi_S(\mathbf{r}_S(x,t),t)|^2 \frac{\partial \mathbf{r}_T^S}{\partial t}\cdot\frac{\partial \mathbf{r}_T^S}{\partial t}\,dxdt \\
& +K_1\int_0^{t_f}\int_\Omega |\phi_N|^2\mathbf{A}\cdot\frac{\partial \mathbf{r}_T^N(x,t)}{\partial t}\,dxdt \\
& +K_1\int_0^{t_f}\int_\Omega |\phi_S|^2\mathbf{A}\cdot\frac{\partial \mathbf{r}_T^S(x,t)}{\partial t}\,dxdt \\
& +\frac{\gamma_N}{2}\int_0^{t_f}\int_\Omega |\nabla\phi_N - iw_N\mathbf{A}\phi_N|^2\,dxdt \\
& +\frac{\gamma_S}{2}\int_0^{t_f}\int_\Omega |\nabla\phi_S - iw_S\mathbf{A}\phi_S|^2\,dxdt \\
& -\frac{(\gamma_0)_N}{2}\int_0^{t_f}\int_\Omega \frac{\partial\phi_N(x,t)}{\partial t}\,\frac{\partial(\phi_N)^*(x,t)}{\partial t}\,dxdt \\
& -\frac{(\gamma_0)_S}{2}\int_0^{t_f}\int_\Omega \frac{\partial\phi_S(x,t)}{\partial t}\,\frac{\partial(\phi_S)^*(x,t)}{\partial t}\,dxdt \\
& +\frac{\alpha_N}{4}\int_0^{t_f}\int_\Omega |\phi_N|^4\,dxdt + \frac{\alpha_S}{4}\int_0^{t_f}\int_\Omega |\phi_S|^4\,dxdt \\
& +J_{Aux_1}+J_{Aux_2}+J_{Aux_3}+J_{Aux_4},
\end{aligned} \tag{43}
$$

where

$$
\begin{aligned}
J_{Aux_1} \;=\; & \int_0^{t_f}\int_\Omega \lambda_1\left(\mathbf{v}_N - \frac{\partial \mathbf{r}_N(x,t)}{\partial t}\right)\,dxdt \\
& +\int_0^{t_f}\int_\Omega \lambda_2\left(\mathbf{v}_S - \frac{\partial \mathbf{r}_S(x,t)}{\partial t}\right)\,dxdt \\
& +\int_0^{t_f}\int_\Omega \hat{P}\left(\frac{d\rho}{dt} + \rho_N \text{ div } \mathbf{v}_N + \rho_S \text{ div } \mathbf{v}_S\right)\,dxdt,
\end{aligned} \tag{44}
$$

where we recall that

$$P = P(\hat{P},\rho) = \frac{d(\hat{P}\rho)}{dt},$$

$$P_N = \frac{d(\hat{P}\rho_N)}{dt},$$

$$P_S = \frac{d(\hat{P}\rho_S)}{dt}.$$

Moreover,

$$
\begin{aligned}
& J_{Aux_2} \\
= & -\int_0^{t_f} E(t)\left(\int_\Omega \left(|\phi_N(\mathbf{r}_N(x,t),t)|^2+|\phi_N(\mathbf{r}_N(x,t),t)|^2\right)dx-m_T\right)\,dt \quad (45)
\end{aligned}
$$

$$
\begin{aligned}
& J_{Aux_3} \\
= & \int_0^{t_f}\int_\Omega \lambda_3\left(C_{v_N}\rho_N\frac{dT_N}{dt}+C_{v_S}\rho_S\frac{dT_S}{dt}+P_N\ \text{div}\ (\mathbf{v}_N)+P_S\ \text{div}\ \mathbf{v}_S\right) \\
& -K_N\nabla^2T_N-K_S\nabla^2T_S-\left.\frac{\partial Q}{\partial t}\right)\,dxdt \\
& +\int_0^{t_f}\int_\Omega \lambda_4\left(C_{v_N}T_N-\frac{1}{2}|\phi_N(\mathbf{r}_N(x,t),t)|^2\frac{\partial\hat{\mathbf{r}}_N}{\partial t}\cdot\frac{\partial\hat{\mathbf{r}}_N}{\partial t}\right)\,dxdt \\
& +\int_0^{t_f}\int_\Omega \lambda_5\left(C_{v_S}T_S-\frac{1}{2}|\phi_S(\mathbf{r}_S(x,t),t)|^2\frac{\partial\hat{\mathbf{r}}_S}{\partial t}\cdot\frac{\partial\hat{\mathbf{r}}_S}{\partial t}\right)\,dxdt, \qquad (46)
\end{aligned}
$$

and

$$
\begin{aligned}
J_{Aux_4} = & -\int_0^{t_f}\int_\Omega K_1V(\{\mathbf{r}_N\},\{\mathbf{r}_S\})\left(|\phi_N|^2+|\phi_S|^2\right)\,dxdt \\
& +K_1\int_0^{t_f}\int_\Omega |\phi_N|^2\mathbf{B}(x,t)\times\frac{\partial\mathbf{r}_N(x,t)}{\partial t}\cdot(\mathbf{r}_N(\varepsilon t,x)-(\mathbf{r}_0)_N(x))\,dxdt \\
& +K_1\int_0^{t_f}\int_\Omega |\phi_S|^2\mathbf{B}(x,t)\times\frac{\partial\mathbf{r}_S(x,t)}{\partial t}\cdot(\mathbf{r}_S(\varepsilon t,x)-(\mathbf{r}_0)_S(x))\,dxdt \\
& +\frac{1}{8\pi}\int_0^{t_f}\int_\Omega |\ \text{curl}\ \mathbf{A}-\mathbf{B}_0|^2\,dxdt \\
& +\frac{1}{8\pi}\int_0^{t_f}\int_\Omega \left|\nabla V(\mathbf{r}_N,\mathbf{r}_S)+\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right|^2\,dxdt, \qquad (47)
\end{aligned}
$$

where

$$\mathbf{B}=\ \text{curl}\ \mathbf{A}-\mathbf{B}_0$$

is the total magnetic field, $\mathbf{A}=(A_1,A_2,A_3)$ is the magnetic potential,

$$V(\mathbf{r}_N,\mathbf{r}_S\})$$

is the electric potential and

$$\mathbf{E}=-\nabla V(\mathbf{r}_N,\mathbf{r}_S\})-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},$$

is the electric field.

Remark 4.2. *As a final remark, about the domain of the functional J_7 we have not so far addressed it explicitly. However for such a domain we would specify sets of generically smooth C^2 class functions defining only the initial conditions*

$$\{\mathbf{r}_N\}|_{t=0} = \{(\mathbf{r}_0)_N\},$$

$$\{\mathbf{r}_S\}|_{t=0} = \{(\mathbf{r}_0)_S\}$$

and $T(x,0) = T_0$, considering the remaining boundary and initial conditions just as natural ones.

Remark 4.3. *The variation of J_7 in $\mathbf{r}_N$ which lead us to the correct Euler-Lagrange equation is defined by*

$$\begin{aligned}
& \hat{\delta}(J_7)_{\mathbf{r}_N}(\mathbf{r}_N, \mathbf{r}_S, \hat{\mathbf{r}}_N, \hat{\mathbf{r}}_S, \mathbf{r}_N(\varepsilon t, x), \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, T, \hat{P}, \mathbf{A}, V, E, \lambda; \varphi) \\
= \; & \lim_{h \to 0} \Big[J_7(\mathbf{r}_N + h\varphi, \mathbf{r}_S, \hat{\mathbf{r}}_N, \hat{\mathbf{r}}_S, \mathbf{r}_N(\varepsilon t, x) + h\varphi, \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, T, \hat{P}, \mathbf{A}, V, E, \lambda) \\
& -J_7(\mathbf{r}_N, \mathbf{r}_S, \hat{\mathbf{r}}_N, \hat{\mathbf{r}}_S, \mathbf{r}_N(\varepsilon t, x), \mathbf{r}_S(\varepsilon t, x), \phi_N, \phi_S, T, \hat{P}, \mathbf{A}, V, E, \lambda) \Big] / h \\
= \; & \mathbf{0}, \; \forall \varphi \in C_c^\infty(\Omega \times [0, t_f]; \mathbb{R}^3).
\end{aligned} \tag{48}$$

The corresponding Euler-Lagrange equation stands for

$$\begin{aligned}
& \frac{\partial}{\partial t}\left(|\phi_N|^2 \frac{\partial \mathbf{r}_T^N(x,t)}{\partial t} \right) - K_1 \frac{\partial}{\partial t}\left(|\phi_N|^2 \mathbf{A} \right) \\
& K_1 \frac{\partial V}{\partial \mathbf{r}_N}(|\phi_N|^2 + |\phi_S|^2) + K_2 |\phi_N|^2 \mathbf{B} \times \frac{\partial \mathbf{r}_N}{\partial t} + \mathcal{O}(\varepsilon) \\
& -\frac{1}{4\pi} \, div \, \mathbf{E} \frac{\partial V}{\partial \mathbf{r}_N} + \frac{\partial \lambda_1}{\partial t} = \mathbf{0}.
\end{aligned} \tag{49}$$

Considering a Maxwell equation in question and letting $\varepsilon \to 0$, we obtain

$$\begin{aligned}
& \frac{\partial}{\partial t}\left(|\phi_N|^2 \frac{\partial \mathbf{r}_T^N(x,t)}{\partial t} \right) - K_1 \frac{\partial}{\partial t}\left(|\phi_N|^2 \mathbf{A} \right) \\
& + K_1 |\phi_N|^2 \mathbf{B} \times \frac{\partial \mathbf{r}_N}{\partial t} + \frac{\partial \lambda_1}{\partial t} = \mathbf{0}.
\end{aligned} \tag{50}$$

We highlight the solution of the concerned system including such an Euler-Lagrange equation is computable, through the Newton's method for example, by expanding J_7 as a quadratic approximate function in φ about $(\mathbf{r}, \mathbf{r}(\varepsilon t, x))$ in each iteration.

A similar remark is valid for the variation of J_7 in $\mathbf{r}_S$ and for the functionals J_5.

Remark 4.4. *The variation of J_7 in* $\mathbf{A}$ *stands for*

$$\begin{aligned}
& -\ curl\ (\ curl\ \mathbf{A}-\mathbf{B}_0)-\frac{1}{c}\frac{\partial}{\partial t}\left(\nabla V(\mathbf{r}_N,\mathbf{r}_S)+\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right) \\
& +\mathcal{O}(\varepsilon)+4\pi K_1|\phi_N|^2\frac{\partial \mathbf{r}_N}{\partial t}++4\pi K_1|\phi_S|^2\frac{\partial \mathbf{r}_S}{\partial t} \\
= \ & \mathbf{0}.
\end{aligned} \tag{51}$$

Letting $\varepsilon \to 0$, *we have the following Maxwell equation*

$$-\ curl\ \mathbf{B}+\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}+4\pi\tilde{J}=\mathbf{0},$$

where

$$\tilde{J}=K_1|\phi_N|^2\frac{\partial \mathbf{r}_N}{\partial t}++K_1|\phi_S|^2\frac{\partial \mathbf{r}_S}{\partial t}.$$

The variation of J_7 in V stands for

$$-4\pi K_1(|\phi_N|^2+|\phi_S|^2)-\ div\ \left(\nabla V(\mathbf{r}_N,\mathbf{r}_S)+\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right)=\mathbf{0},$$

so that we obtain another Maxwell equation

$$div\ \mathbf{E}=4\pi K_1(|\phi_N|^2+|\phi_S|^2).$$

Moreover, from

$$\mathbf{B}=\ curl\ \mathbf{A}-\mathbf{B}_0$$

and assuming $div\ (\mathbf{B}_0)=0$, *we have got*

$$div\ \mathbf{B}=\ div\ (\ curl\ \mathbf{A})=0,$$

which is a third Maxwell equation.

Finally, the fourth Maxwell equation is obtained recalling that

$$\mathbf{E}=-\nabla V(\mathbf{r}_N,\mathbf{r}_S)-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t},$$

so that

$$curl\ \mathbf{E}=-\ curl\ (\nabla V)-\frac{1}{c}\ curl\ \left(\frac{\partial \mathbf{A}}{\partial t}\right),$$

and therefore,

$$curl\ \mathbf{E}+\frac{1}{c}\left(\frac{\partial\ curl\ \mathbf{A}}{\partial t}\right)=\mathbf{0},$$

that is,

$$curl\,\mathbf{E}+\frac{1}{c}\left(\frac{\partial(\mathbf{B}+\mathbf{B}_0)}{\partial t}\right)=\mathbf{0}.$$

Thus, if $\mathbf{B}_0$ *is time independent, that is,*

$$\frac{\partial \mathbf{B}_0}{\partial t}=\mathbf{0},$$

we have

$$curl\,\mathbf{E}+\frac{1}{c}\left(\frac{\partial \mathbf{B}}{\partial t}\right)=\mathbf{0}.$$

In summary, we have got all the four Maxwell equations as necessary conditions for a extremal point of J_7.

5 Conclusion

In this article we have developed a variational formulation for modeling a chemical reaction through a fluid motion in an Eulerian context, including a scalar temperature field.

We highlight the number $N_T(t)$ is in general very large so that tools of statistical physics may be a possibility to address such a high magnitude for $N_T(t)$. Anyway, we believe the internal energy variables may be appropriately modeled in such a more general quantum context.

Through an appropriate variation of J in $\hat{P}$, we obtain an additional term involving the Lagrange multiplier λ_1 in the continuity equation. We believe such a mentioned part results from the effect of the internal energy variables on the mass conservation equation.

Finally, in the last sections, we have presented variational formulations for models in superconductivity. Such models are based on the standard Ginzburg-Landau one combined with an eigenvalue formulation corresponding to a total system mass constraint.

References

[1] P. CONSTANTIN, C. FOIAS, *Navier-Stokes equations,* University of Chicago Press, Chicago, 1989.

[2] C. De Lellis, E. Brué, D. Albritton, M. Colombo, V. Giri, M. Janisch and H. Kwon, *Instability and Non-uniqueness for the 2D Euler Equations,* after M. Vishik (AMS-219) Volume 219 in the series Annals of Mathematics Studies, Princeton University Press, New Jersey, 2024. https://doi.org/10.1515/9780691257846

[3] F. Botelho, *Variational Formulations for the Euler System in Fluid Mechanics and Related Models.* Authorea. July 31, 2024. DOI: 10.22541/au.172243894.42056083/v1

[4] M. Hamouda, D. Han, C-Y. Jung, R. Temam, *Boundary layers for the 3D primitive equations in a cube: the zero-mode,* Journal of Applied Analysis and Computation. 2018; 8, No. 3, (873-889), DOI:10.11948/2018.873.

[5] A. Giorgini, A. Miranville, R. Temam, *Uniqueness and regularity for the Navier-Stokes-Cahn-Hilliard system,* SIAM J. of Mathematical Analysis (SIMA).2019; 51, 3, (2535-2574), https://doi.org/10.1137/18M1223459.

[6] C. Foias, R.M. Rosa, R. Temam R, *Properties of stationary statistical solutions of the three-dimensional Navier-Stokes equations,* J. of Dynamics and Differential Equations, Special issue in memory of George Sell. 2019; 31, 3, 1689-1741, https://doi.org/10.1007/s10884-018-9719-2.

[7] R. Temam, *Navier-Stokes Equations,* AMS Chelsea, reprint (2001).

[8] J.C. Strikwerda, *finite difference cchemes and partial differential equations*, second edition Philadelphia, SIAM, 2004.

[9] F.S. Botelho, *Approximate numerical procedures for the Navier-Stokes system through the generalized method of lines,* Nonlinear Engineering, vol. 13, no. 1, 2024, pp. 20220376. https://doi.org/10.1515/nleng-2022-0376

[10] F. Botelho, *Existence of solution for the Ginzburg-Landau system, a related optimal control problem and its computation by the generalized method of lines*, Applied Mathematics and Computation, 2012; 218 (11976-11989).

[11] F.S. Botelho, *An approximate proximal numerical procedure concerning the generalized method of lines.* Mathematics 2022, 10(16), 2950; https://doi.org/10.3390/math10162950.

[12] F. Botelho *Functional Analysis and Applied Optimization in Banach Spaces*, (Springer Switzerland, 2014).

[13] F.S. Botelho, *Functional Analysis, Calculus of Variations and Numerical Methods for Models in Physics and Engineering,* CRC Taylor and Francis, Florida, 2020.

[14] F.S. Botelho, *Advanced Calculus and its Applications in Variational Quantum Mechanics and Relativity Theory,* CRC Taylor and Francis, Florida, 2021.

[15] R.A. Adams RA and J.F. Fournier, *Sobolev Spaces*, 2nd edn. (Elsevier, New York, 2003).

[16] J.C. Tannehill, D.A. Anderson, R.H. Pletcher, *Computational Fluid Mechanics and Heat Transfer,* 2nd edn., Taylor and Francis, Philadelphia, PA, 1997.

[17] J.F. Annet, *Superconductivity, Superfluids and Condensates*, Oxford Master Series in Condensed Matter Physics, Oxford University Press, Reprint (2010)

[18] L.D. Landau and E.M. Lifschits, *Course of Theoretical Physics, Vol. 5- Statistical Physics, part 1*, Butterworth-Heinemann, Elsevier, New York, reprint (2008).

[19] E.M. Lifschits and L.P. Pitaevskii, *Course of Theoretical Physics, Vol. 9- Statistical Physics, part 2*, Butterworth-Heinemann, Elsevier, New York, reprint (2002).

ISSN: 0162-5519

Established in 1978 by Prof. R. M. Santilli at Harvard University Hadronic Journal and Algebras, Groups and Geometries have been regularly published since 1978 without publication charges and are among the few remaining independent refereed journals.

This Journal publishes advances research papers and Ph. D. theses in any field of mathematics without publication charges.

For subscription, format and any other information please visit the website http://www.hadronicpress.com

HADRONIC PRESS INC.
35246 U. S. 19 North Suite 215
Palm Harbor, FL 34684, U.S.A.
http://www.hadronicpress.com
Email: info@hadronicpress.com
Phone: +1-727-946-0427

HADRONIC JOURNAL Volume 48, Number 1, March 2025

HADRONIC PRESS, INC.